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EFFECT OF LEG SEGMENTATION ON THE EFFICIENCY OF PERMEABLE THERMOELEMENT OF *Co-Sb* BASED MATERIALS

The results of computer research on the effect of leg segment length of Co-Sb-based material on energy conversion efficiency are presented. The optimal operating modes of a 2-segment thermoelement are determined whereby the maximum efficiency values are realized. The possibility of 1.1-1.2-fold increase of the electric power generated by using 2-segment permeable thermoelements of Co-Sb-based materials is demonstrated. Bibl. 14, Fig. 2.

Key words: permeable thermoelements, segmented materials, computer design, Co-Sb based materials.

Introduction

This paper contributes to further research started in [1] and is dedicated to the use of thermal waste by means of permeable segmented thermoelements. It is known that the use of segmented legs in thermoelements allows increasing the efficiency and the generated electric power [2]. This is achieved both due to expansion of the operating temperature range at leg segmentation, and due to the choice of materials with maximum figure of merit value over the entire temperature range. At the same time, in the course of recent decades, the researchers' attention has been drawn to promising materials, namely *Co-Sb* based scutterudites [3], which hold much promise for being used in high-temperature legs of thermoelectric generators in the temperature range up to 800 K. They are characterized by rather high values of the Seebeck coefficient and electrical conductivity. Maximum ZT values of such materials are 1-1.1 at a temperature of 700 K [4 – 5].

However, studies on the use of permeable thermoelements of *Co-Sb* based segmented materials have not been encountered in the literature. Therefore, the purpose of the work is to determine the characteristics of permeable segmented thermoelements of *Co-Sb* based materials in electric energy generation mode, as well as to find their optimal operating conditions and parameters whereby maximum efficiency of thermal into electric energy conversion is realized.

Physical model and its mathematical description

A physical model of permeable thermoelement in electric energy generation mode is represented in Fig. 1. The thermoelement consists of n- and p-type legs whose physical properties are temperature-dependent. Heat input is realized by passing heat carrier along the legs through the channels (pores). Each leg comprises N_n and N_p segments, and the contact resistance of compound is r_0. The lateral surfaces of the legs are adiabatically isolated; heat carrier temperature at thermoelement inlet T_m is assigned. The temperature of cold junctions T_c is thermostated.

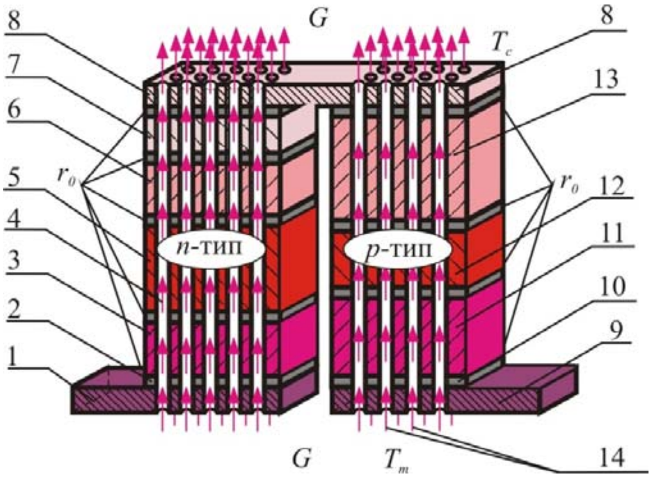

Fig. 1. Physical model of permeable segmented thermoelement.
1, 8, 9 – connecting plates; 2, 10 – connecting layers;
3, 5, 6, 7 – segments of n-type leg; 4,14 – heat carrier;
11, 12, 13 – segments (sections) of p-type leg.

A system of differential equations describing the distribution of temperatures and heat fluxes in a steady-state one-dimensional case, in the infinitely small part *dx* of each *k*-th segment of *n*- and *p*-type legs, in the dimensionless coordinates is given by relations [7]:

$$\left.\begin{aligned} &\frac{dT}{dx} = -\frac{\alpha_k j}{\kappa_k}\,T - \frac{j}{\kappa_k}q, \\ &\frac{dq}{dx} = \frac{\alpha_k^2 j}{\kappa_k}T + \frac{\alpha_k j}{\kappa_k}q + j\rho_k + \frac{\alpha_T P_K^1 N_K l_K^2}{\left(S - S_K\right)j}\left(t - T\right), \\ &\frac{dt}{dx} = \frac{\alpha_T P_K^1 N_K l_k}{Gc_P}\left(t - T\right), \end{aligned}\right\}_{n,p} \quad \begin{aligned} &k = 1,\ldots,N_{n,p} \\ &x_{k-1} \le x \le x_k \end{aligned} \tag{1}$$

where $Ï^{1}_{Ê}$ is channel perimeter; N_K is the number of channels, S_K is cross-sectional area of all the channels, S is a section of leg together with the channels, G is heat carrier consumption in the channels, c_P is specific heat of heat carrier, t is heat carrier temperature at point x, T is leg temperature at point x, α_T is heat-transfer coefficient, α, κ, ρ are the Seebeck coefficient, thermal conductivity and resistivity of leg material.

Specific heat fluxes q and the reduced density of electric current j are determined through

$$q = \frac{Q}{I}, \qquad j = \frac{Il}{S}, \tag{2}$$

where Q is power of heat flux passing through thermoelement leg, I is electric current, S is cross-sectional area of thermoelement legs, l – the height of the branch.

The boundary conditions necessary for solving (1) with regard to the Joule-Lenz heat release due to contact resistance r_0 at points of connection of leg segments are formulated as:

$$\begin{aligned} &T_{n,p}\left(0\right) = T_C, \qquad t_{n,p}\left(1\right) = T_m, \qquad q_{n,p}\left(1\right) = 0, \\ &T_{n,p}\left(x_k^+\right) = T_{n,p}\left(x_k^-\right), \qquad q_{n,p}\left(x_k^+\right) = q_{n,p}\left(x_k^-\right) + \frac{r_0}{S_{n,p}}I, \end{aligned} \tag{3}$$

where indices "-" and "+" denote the values of functions immediately to the left and right of the interface of segments x_k; $k = 1, \ldots, N$ is the index which determines leg segment number.

For seeking optimal values of doping impurities which determine carrier concentrations in leg segments it is necessary to assign the dependences of material parameters α, κ, ρ on temperature and concentration of carriers (or impurities). The main task in the design of permeable segmented generator thermoelement is to determine such agreed parameters (reduced current density j in the legs, heat carrier consumption in channels G, concentration of doping impurities in materials of each segment) whereby the efficiency of the thermoelement reaches a maximum.

The efficiency will be determined through the relation of electric power P generated by the thermoelement to a change in heat carrier enthalpy:

$$\eta = \frac{P}{\sum\limits_{n,p} Gc_p\left(T_m - T_C\right)} . \tag{4}$$

The maximum efficiency can be conveniently reduced to achievement of functional minimum:

$$J = \ln\left[\sum_{n,p}\left\{Gc_P\left(T_m - T_C\right)\right\}\right] - \ln\left[\sum_{n,p}\left\{Gc_P\left(T_m - t(0)\right) + q(0)\frac{j\left(S - S_K\right)}{l} - I\left(\frac{r_0}{S_n} + \frac{r_0}{S_p}\right)\right\}\right]. \tag{5}$$

This problem was solved through use of the Pontryagin maximum principle [8], yielding the necessary optimality conditions:

1. Optimal values of specific current density in thermoelement legs j must satisfy the equalities

$$-\left[\frac{\partial J}{\partial j}\right]_{n,p} + \sum_{n,p}\int_0^1\left[\psi_1^k\frac{\partial f_1^k}{\partial j_k} + \psi_2^k\frac{\partial f_2^k}{\partial j_k} + \psi_3^k\frac{\partial f_3^k}{\partial j_k}\right]_{n,p} dx = 0 , \tag{6}$$

where $\left(f_1^k, f_2^k, f_3^k\right)_{n,p}$ are right-hand sides of equations (1), $\psi = \left(\psi_1^k, \psi_2^k, \psi_3^k\right)_{n,p}$ is pulse vector function [3, 4], which is found from solving the additional system of differential equations

$$\left.\begin{aligned}
&\frac{d\psi_1}{dx} = \frac{\alpha_k j_k}{\kappa_k}R_1\psi_1 - \left(\frac{\alpha_k j_k}{\kappa_k}R_2 - \frac{\alpha_e l_k}{\left(S - S_{\hat{E}}\right)j_k}\right)\psi_2 + \frac{\alpha_T \ddot{I}_K^{\,1} N_K}{Gc_P}\psi_3,\\
&\frac{d\psi_2}{dx} = \frac{j_k}{\kappa_k}\psi_1 - \frac{\alpha_k j_k}{\kappa_k}\psi_2,\\
&\frac{d\psi_3}{dx} = -\frac{\alpha_T \ddot{I}_K^{\,1} N_K l_k}{\left(S - S_{\hat{E}}\right)j_k}\psi_2 - \frac{\alpha_T \ddot{I}_K^{\,1} N_K}{Gc_P}\psi_3,
\end{aligned}\right\}_{n,p} \tag{7}$$

where

$$\left.\begin{aligned}
&R_1 = 1 + \frac{d\ln\alpha}{dT}T - \frac{d\ln\kappa}{dT}\left(T + \frac{q}{\alpha}\right),\\
&R_2 = R_1 + \frac{\kappa}{\alpha^2\sigma}\frac{d\ln\sigma}{dT} + \frac{d\ln\kappa}{dT}\left(T + \frac{q}{\alpha}\right).
\end{aligned}\right\}_{n,p}$$

with the boundary conditions

$$\psi_1^{n,p}(1)=0,$$

$$\psi_2^{n,p}(0)=\frac{\dfrac{j(S-S_K)}{l}}{\sum\limits_{n,p}\left\{Gc_P\left(T_m-t(0)\right)+q(0)\dfrac{j(S-S_K)}{l}-I\left(\dfrac{r_0}{S_n}+\dfrac{r_0}{S_p}\right)\right\}}, \tag{8}$$

$$\psi_3^{n,p}(0)=\frac{Gc_P}{\sum\limits_{n,p}\left\{Gc_P\left(T_m-t(0)\right)+q(0)\dfrac{j\ (S-S_K)}{l}-I\left(\dfrac{r_0}{S_n}+\dfrac{r_0}{S_p}\right)\right\}}.$$

2. Optimal values of heat carrier consumption G in the channels

$$-\left[\frac{\partial J}{\partial G}\right]_{n,p}+\sum_{n,p}\int_0^1\left[\psi_1^k\frac{\partial f_1^k}{\partial G}+\psi_2^k\frac{\partial f_2^k}{\partial G}+\psi_3^k\frac{\partial f_3^k}{\partial G}\right]_{n,p}dx=0\,. \tag{9}$$

3. Optimal values of doping impurities in each segment material C_k are found from the relations

$$\int_0^1\left[\psi_1^k\frac{\partial f_1^k}{\partial C_k}+\psi_2^k\frac{\partial f_2^k}{\partial C_k}+\psi_3^k\frac{\partial f_2^k}{\partial C_k}\right]_{n,p}dx=0, \qquad k=1\ldots,N_{n,p}. \tag{10}$$

In the case of thermoelement design for fixed segment materials the optimality conditions (10) are disregarded. Such a method as applied to thermoelectric power conversion is described in many works, for instance [2, 9], and was used for creation of computer program and research on permeable segmented thermoelement of *Co-Sb* based materials.

Results of computer research on permeable segmented generator thermoelement based on *Co-Sb*

Results of calculations of the efficiency and electric power dependence on leg segment height are represented in Fig.2. This figure shows the dependences for 1- and 2-segment legs of permeable thermoelement for different heat carrier temperatures at the inlet to thermoelement. The data is given for the case when the cross-sectional area of the leg together with the channels was $S = 1$ cm^2, the contact resistance at point of connection of legs was $r_0 = 5\cdot10^{-6}$ Ωm·cm^2. The calculation was made under thermostating of cold junctions at a temperature of T_c=300 K for different values of heat carrier temperature at the inlet to thermoelement T_m = 900 K, 1100 K, 1500 K. In so doing, the temperature of thermoelement hot junctions was program controlled, so as not to exceed 800 K – the boundary value of the temperature dependences of *Co-Sb* based materials.

As a result of calculations, the optimal values j, G and doping parameter x were found whereby maximum efficiency of thermal into electric energy conversion is realized. The dependences of the efficiency and power on the leg height were determined.

The dependence of maximum efficiency η and specific electric power W of permeable generator thermoelement at optimal values of j, G and doping parameter x of legs on the leg height l_k for different heat carrier temperatures is shown in Fig. 2. The data is given for channel diameter d_k = 0.1 cm and the number of channels $N_k = 25$ pcs per 1 cm^2.

It is seen that the generated electric power has a maximum at leg segment height which is in the area of 1.0-1.6cm. A gain in electric power with the use of a 2-segment leg can be 10-20 %. No essential efficiency gain with the use of a 2-segment leg is observed at the value of contact resistance $r_0 = 5\cdot10^{-6}$ Ωm·cm^2.

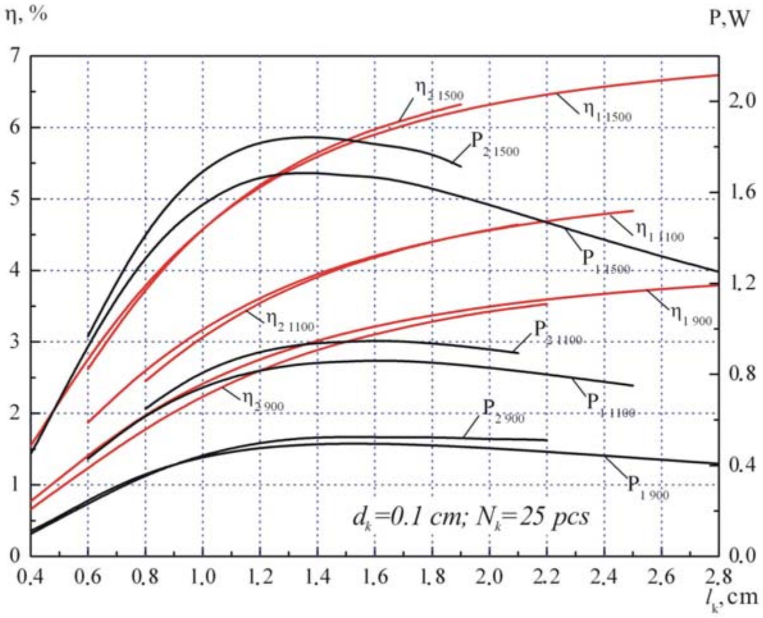

Fig.2. Dependence of efficiency and specific electric power W of permeable generator thermoelement on leg segment height l_k.

This is due to the negative influence of the Joule-Lenz heat at points of connection of leg segments. Higher efficiency values can be achieved with a decrease in contact resistance at points of connection of leg segments.

Conclusions

1. The effect of leg segmentation in permeable thermoelement of *Co-Sb* based materials on the basic power conversion characteristics was determined. Optimal parameters of a 2-segment permeable thermoelement whereby maximum power conversion efficiency is realized were found.
2. It is shown that the electric power generated by a 2-segment permeable thermoelement of *Co-Sb* based materials can be 1.1-1.2 fold higher.

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ВПЛИВ СЕГМЕНТУВАННЯ ВІТОК НА ЕФЕКТИВНІСТЬ ПРОНИКНОГО ТЕРМОЕЛЕМЕНТА З МАТЕРІАЛІВ НА ОСНОВІ *Co-Sb*

Приведено результати комп'ютерних досліджень впливу довжини сегментів віток з матеріалів на основі Co-Sb на ефективність перетворення енергії. Визначено оптимальні режими роботи 2-сегментного термоелемента, за якої реалізуються максимальні значення ККД. Показано можливість покращення електричної потужності, що генерується при використанні 2-сегментних проникних термоелементів з матеріалів на основі Co-Sb, в 1.1-1.2 рази. Бібл. 14, рис. 2.
Ключові слова: проникні термоелементи, сегментні матеріали, комп'ютерне проектування, матеріали на основі Co-Sb.

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ВЛИЯНИЕ СЕГМЕНТИРОВАНИЯ ВЕТВЕЙ НА ЭФФЕКТИВНОСТЬ ПРОНИЦАЕМОГО ТЕРМОЭЛЕМЕНТА ИЗ МАТЕРИАЛОВ НА ОСНОВЕ *Co-Sb*

Приведены результаты компьютерных исследований влияния длины сегментов ветвей из материалов на основе Co-Sb на эффективность преобразования энергии. Определены оптимальные режимы работы 2- сегментного термоэлемента, по которым реализуются максимальные значения КПД. Показана возможность улучшения электрической мощности, которая генерируется при использовании 2-сегментных проницаемых термоэлементов из материалов на основе Co-Sb, в 1.1-1.2 раза. Библ. 14, рис. 2.

Ключевые слова: *проницаемые термоэлементы, сегментные материалы, компьютерное проектирование, материалы на основе Co-Sb.*

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