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DAMPING OF THERMOMECHANICAL STRESSES AS A MEANS OF INCREASING THE CYCLIC STABILITY OF THERMOELECTRIC ENERGY CONVERTERS

Based on a combination of the strength of materials methods with the Weibull approach, the requirements for the rigidity coefficients of damping elements are determined, which can be used to reduce thermomechanical stresses in thermoelectric legs in order to increase the cyclic stability of thermoelectric energy converters. The Coffin-Manson power model for the dependence of the acceleration factor on the temperature difference in the presence of cyclic temperature effects is substantiated. The calculation results are not only in qualitative but also in satisfactory quantitative agreement with the experimental data.
Key words: cyclic stability, thermoelectric energy converter, thermomechanical stresses, damping, strength of materials, Weibull approach, cracking strength, rigidity of elastic element.

Introduction

The need for damping of thermomechanical stresses arises from the fact that in the case of rigid fastening of thermoelectric legs to ceramic plates in the presence of cyclic temperature effects, thermomechanical stresses arise in thermoelectric legs, which significantly exceed their cracking resistance and significantly reduce the probability of failure-free operation of thermoelectric energy converters [1]. The purpose of the article is to determine the requirements for the rigidity of damping elements in order to obtain mechanical stresses in thermoelectric legs that are acceptable from the point of view of cyclic stability and the probability of failure-free operation of thermoelectric energy converters.

1. Physical model of thermomechanical stress damping and its consequences

Physical model of thermomechanical stress damping is shown in Fig. 1.

To determine the requirements for the rigidity coefficient of an elastic element, let us imagine that the thermoelectric leg expands partially freely. Its temperature deformation by the value x causes the reaction force kx from the elastic element. This reaction force corresponds to the mechanical stress kx/b^2 , where b is the cross-sectional side of the leg. The following equation follows from the condition of mechanical equilibrium for determining x:

$$E\left(\alpha^T \Delta T - \frac{x}{l}\right) = \frac{kx}{b^2}. \quad (1)$$

where E is the Young's modulus of the thermoelectric material, α_T is the coefficient of its linear expansion, ΔT is the temperature difference across the thermoelectric leg, l is its length.

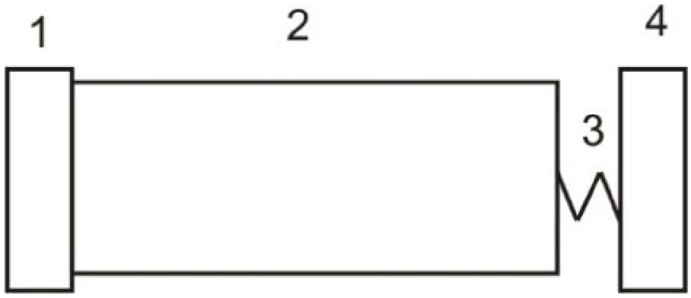

Fig. 1. Physical model of damping of a thermoelectric leg on the hot side,
1 – rigidly fastened ceramics on the cold side,
2 – thermoelectric leg with anti-diffusion layers and connections,
3 – elastic element; 4 – ceramics on the hot side.

Solving (1) and taking into account the generalized Hooke's law, we find the following expression for the residual mechanical stress that causes cracking of the thermoelectric leg in the presence of a damping element:

$$\sigma_f = \frac{kEl\alpha^T \Delta T}{\left(Eb^2 + kl\right)\left(1-\nu\right)} \quad (2)$$

where ν is the Poisson's ratio of the thermoelectric material.

It is clear that if the elastic element is perfectly compliant, then $k = 0$, the expansion is free and, therefore, there are no destructive stresses, and if we have a perfectly rigid fastening, then $k \to \infty$ and we get the traditional formula for the destructive stress of cracking under thermomechanical loads.

It is this stress that should be substituted into the Weibull distribution when calculating the probability of failure-free operation of a thermoelectric generator module in the presence of cyclic temperature effects. Taking into account the presence of a temperature gradient along the leg and approximately neglecting the temperature dependence of the thermal conductivity of the material, we obtain the following expression for the probability of failure-free operation of the module in the presence of cyclic temperature effects:

$$P\left(N_c\right) = \exp\left\{-\frac{2N_c N_L b^2 l}{m+1}\left(\frac{kEl\alpha^T \Delta T}{\left(Eb^2 + kl\right)\left(1-\nu\right)\sigma_0}\right)^m\right\}, \quad (3)$$

where N_c is the number of heating-cooling cycles, N_L is the number of legs in the module, m and σ_0 are the Weibull parameters of the thermoelectric material. And from formula (3) it is possible to determine such reliability indicators as the average cyclic stability and the percentage resource of cyclic stability γ. The results of these calculations are shown in Figs. 2, 3.

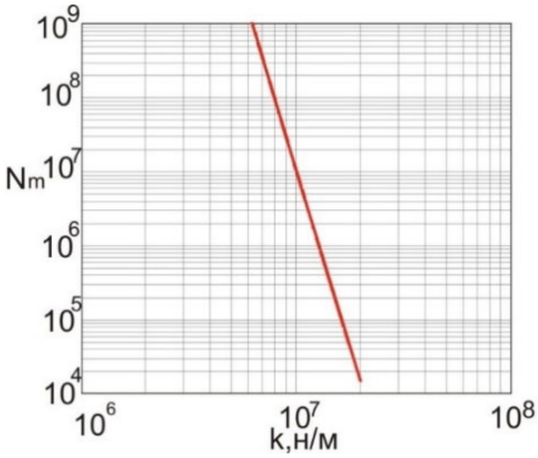

Fig. 2. Predicted dependence of the average cyclic stability of thermoelectric generator modules on the rigidity coefficient of the elastic element.

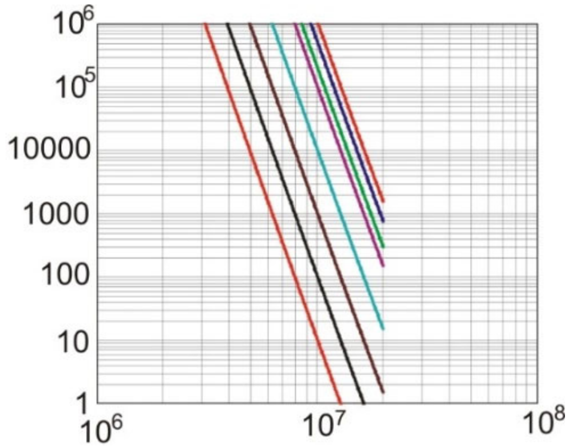

Fig. 3. Predicted dependence of gamma-percentage cyclic stability of thermoelectric generator modules on the rigidity coefficient of the elastic element in N/m. The lines from left to right correspond to values of γ equal to 0.999999, 0.99999, 0.9999, 0.9999, 0.99, 0.98, 0.95, 0.9.

In practice, stress relaxation is achieved either by proper selection of solders [2], or by installing special gaskets between the ceramic and copper connections [1].

It is evident from the figures that in going from absolutely rigid fastening of thermoelectric legs to fastening of albeit very high but finite rigidity, the predicted cyclic stability of thermoelectric energy converters grows very rapidly. This rapid growth is primarily due to the large value of the shape parameter *m*.

But this at first glance encouraging result is in fundamental contradiction with the approaches that have been formed and generally accepted within the disciplines of "Strength of Materials" and "Structural Mechanics". For these disciplines, the key characteristic of a material is its ultimate strength. And the Weibull probabilistic approach does not provide for such a characteristic. Therefore, within its framework, even at arbitrarily large thermomechanical stresses, there remains a probability, albeit very small, but significantly different from zero, of preserving the integrity of the thermoelectric leg. But from the basic principles of strength of materials and structural mechanics it follows that, regardless of the cyclic stability index, the “acceptable” rigidity coefficient of the damping element should be determined only by the requirement that the damped thermomechanical stresses do not exceed the minimum cracking strength σ_f of the thermoelectric leg. And according to Griffiths' theory, it is defined as follows [3]:

$$\sigma_f = \frac{K_c}{\sqrt{\pi l}} \tag{4}$$

where K_c is the so-called load capacity of the thermoelectric leg material, l is the length of the thermoelectric leg along the temperature gradient. But in this case, it turns out that, for example, the rigidity coefficient of the damping element for a thermoelectric leg based on bismuth telluride in the form of a cube with an edge of 5 mm at a temperature difference of 150 °C should not exceed $1.17\cdot10^7$ n/m. And although this corresponds to the previous probabilistic estimates, it does not yet create a safety margin for crack resistance. There are no recommendations for safety margins in [3] for this case. For example, for a safety factor of 1.5, a rigidity coefficient value of no more than $7.636\cdot10^6$ N/m is required, and for a 10-fold safety factor – a rigidity coefficient value of no more than $1.106\cdot10^6$ N/m. To create a particularly large safety factor, such a damping element can be manufactured, for example, in the form of a miniature spring made of thin wire.

Its rigidity coefficient according to [3] is equal to

$$k = \frac{Ed^4}{16(1+\nu)D^3}, \tag{5}$$

where E and ν are Young's modulus and Poisson's ratio of the spring material, D and d are the diameters of the wire and the spring coil, respectively. From this formula it follows that acceptable damping of thermomechanical stresses is provided, for example, by a single-coil spring with a coil diameter greater than the wire diameter, made of aluminium wire with a diameter of no more than 5.6 mm, and damping with a 10-fold safety margin is provided by the same wire with a diameter of 3 mm with a coil diameter of 5 mm. The rigidity of the damping element is dramatically affected by the diameter of the wire; as a rule, such damping elements are made of wire of a significantly smaller diameter, so the condition of effective damping is well met. And the lower limit of the wire diameter is determined solely by the strength of the connection, and, therefore, the electrical connection of the thermocouples in the energy converter.

In this case, there is no need to worry about the "safe" value of thermal conductivity in terms of mechanical stability, and, consequently, the thermoelectric figure of merit and the efficiency of the thermoelectric material. In this case, the predicted cyclic stability in accordance with Fig. 2 will be no less than 10^7 cycles.

If we evaluate the rigidity coefficient of ceramics from the same point of view, taking into account its Young's modulus and thickness, it turns out to be equal to $2.7\cdot10^9$ N/m.

But elastic elements can also be made of rubber or polymers. Another way to reduce thermomechanical stress is to optimize the geometry of thermoelectric legs, but in itself it still does not provide a level that would guarantee acceptable cyclic stability of generator modules.

At first glance, the predicted cyclic stability and the allowable rigidity of the damping elements seem exaggerated. But it should be taken into account that in this case, the cyclic stability is considered relative to the destruction of thermoelectric legs. However, even without destruction, cyclic temperature effects can change the parameters of the thermoelectric energy converter due to such processes in the thermoelectric material that affect its thermoelectric characteristics. This issue was considered in detail in [4] for materials based on *Si-Ge*, but these processes are not the subject of this study.

The obtained results are at least qualitatively consistent with the results of [5], where it is shown that the highest predicted reliability is obtained when the hot end of the thermoelectric leg is not rigidly fastened, but is pressed, since pressing leads to additional compensation of thermomechanical stresses.

Let us now compare our calculations and estimates of the cyclic stability of thermoelectric energy converters with experimental data from other authors. In [6], accelerated cyclic tests of thermoelectric cooling modules were performed and their failures were analyzed during these tests. Studies have shown that the dependence of the relative number of failures on the number of heating-cooling cycles is described with satisfactory accuracy by Weibull curves. In this case, the shape parameters of the curves do not depend on temperature and are equal to 3.65877, and the scale parameters for temperature differences of 70 and 80 K are equal to 2324.91 and 1830.84, respectively. In this case, the presence of a temperature difference leads to deformation of the thermoelectric legs and bending of the ceramics. Therefore, failures are explained by cracking of the material due to the cyclic action of bending deformations. The probability of failures at the level of 10 % is observed after 1250 cycles at a temperature difference of 70 K and 1000 cycles at a temperature difference of 80 K. In the mentioned work there is no indication of the adoption by the manufacturers of modules of any special measures to reduce thermomechanical stresses.

But formula (3) can also be presented in the form of a Weibull distribution:

$$P\left(N_c\right)=\exp\left(-\frac{N_c}{N_0}\right). \tag{4}$$

The shape parameter of this distribution is 1, and the scale parameter is defined as:

$$N_0=\frac{m+1}{2N_L b^2 l}\left(\frac{\left(Eb^2+kl\right)\left(1-\nu\right)\sigma_0}{kEl\alpha^T\Delta T}\right)^m. \tag{5}$$

Let us introduce some effective temperature of activation of failures associated with the destruction of thermoelectric legs due to cracking, which will be equal to

$$T_{\text{efc}}=\frac{\left(Eb^2+kl\right)\left(1-\nu\right)\sigma_0}{kEl\alpha^T}\left(\frac{m+1}{2N_L b^2 l}\right)^{\frac{1}{m}}. \tag{6}$$

Then the scale parameter can be given as:

$$N_0=\left(\frac{T_{\text{efc}}}{\Delta T}\right)^m. \tag{7}$$

From this point of view, it seems appropriate to introduce a failure acceleration factor through a value inverse to the scale parameter:

$$Af=\left(\frac{\Delta T}{T_{\text{efc}}}\right)^m \tag{8}$$

This representation corresponds to the so-called Coffin-Manson model. According to the general definition of the acceleration factor in this model, it can be represented as follows:

$$AF = \left(\frac{\Delta T_{ALT}}{\Delta T_{nom}} \right)^m \tag{9}$$

where $\Delta T_{ALT}, \Delta T_{nom}$ are temperature differences during accelerated tests and in nominal mode, respectively.

Thus, it turns out that if the cyclic stability of thermoelectric modules is indeed determined by the processes of complete destruction of thermoelectric legs due to cracking under the influence of thermomechanical stresses, then its dependence on the temperature difference should directly characterize the Weibull distribution, which determines the "response" of the thermoelectric material to mechanical stress.

Now let us analyze to what extent such a preliminary conclusion is consistent with the available experimental data. In [6], the value of $m = 1.78914$ was obtained, while according to [7] it should be $m = 10$. Therefore, failures during accelerated cyclic endurance tests are mainly not caused by layer-by-layer cracking of thermoelectric legs as a whole under the influence of thermomechanical stresses. Moreover, the deviation from unity of the Weibull distribution shape parameter obtained during the tests indicates that failures in successive cycles are not independent. Thus, the observed failures depended significantly on the "prehistory" of the modules, which means that there is a gradual growth of fatigue cracks.

In this case, formula (5) can be written as follows:

$$P(N_c) = \exp\left\{ -\frac{2N_c^{\beta} N_L b^2 l}{m+1} \left[\frac{kEl\alpha^T \Delta T}{(Eb^2 + kl)(1-\nu)\sigma_0} \right]^m \right\}. \tag{10}$$

Therefore, taking into account the crack growth process, the scale parameter is now equal to:

$$N_0 = \left\{ \frac{m+1}{2N_L b^2 l} \left[\frac{(Eb^2 + kl)(1-\nu)\sigma_0}{kEl\alpha^T \Delta T} \right]^m \right\}^{\frac{1}{\beta}}, \tag{11}$$

and, hence, expression (9) for the acceleration factor takes the form:

$$AF = \left(\frac{\Delta T_{ALT}}{\Delta T_{nom}} \right)^{\frac{m}{\beta}}. \tag{12}$$

The modules studied in [6] contained 254 legs. This formula reflects the essence of the Coffin-Manson model, which is that the acceleration factor is the temperature difference of the legs connected in series. Considering that the studies were conducted without taking special measures to reduce thermomechanical stresses, assuming the dimensions of the legs to be $b = 3$ mm, $l = 2$ mm and considering σ_0 as a fitting parameter, we obtain that the studied modules at a temperature difference of 150 K could withstand about 106 cycles with a probability of 0.999, if they were given the opportunity to expand almost freely toward the hot side by placing the legs on this side on damping elastic elements with a rigidity coefficient of no more than 106 N/m. The corresponding calculated plot of cyclic stability is shown in Fig. 4.

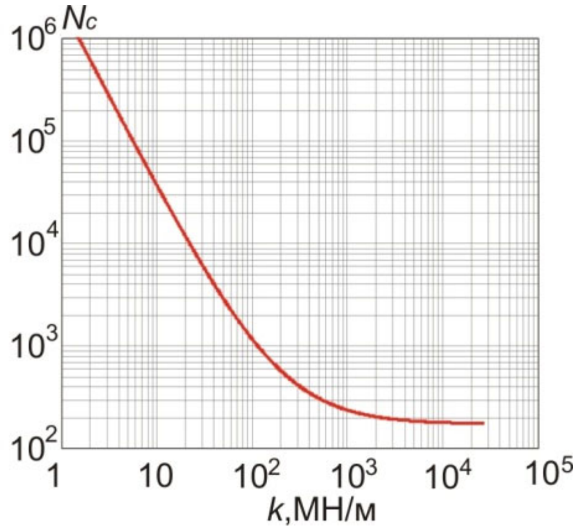

Fig. 4. The predicted cyclic stability of the module with a probability of failure-free operation equal to 0.999 depending on the rigidity coefficient of the elastic element in the presence of cyclic fatigue.

From the comparison of the plots in Figs. 3 and 4 it is seen that in the presence of cyclic fatigue the requirements for the compliance of the elastic element that damps thermomechanical stresses increase almost 10 times. On the other hand, based on the above estimate of ceramics rigidity, made on the basis of data on its Young's modulus and thickness, it can be stated that without additional stress damping this module with a probability of 0.99 will withstand only about 360 cycles at $\Delta T = 150$ °C. Taking into account the acceleration factor, we get that at $\Delta T = 80$ °C it should have withstood 1120 cycles with the same probability, but in reality it withstood 550 cycles. Such a coincidence of theory and experiment can be considered acceptable. Comparison of the prediction with the results of [6] allows us to conclude that in the manufacture of the tested modules special measures for damping thermomechanical stresses were not used. The above comparison of theory with experiment is only approximate and qualitative, since detailed data on the geometric parameters of the modules and the characteristics of the materials used are not provided in [6].

The discrepancy between the prediction results and the experimental data may also be due to the following factors:

1) The Weibull distribution was obtained empirically, its parameters are found exclusively experimentally and it only approximately describes the statistics of failure and cyclic fatigue, and no “first principles” that would allow obtaining more reasonable distributions exist today;

2) as a result, the Coffin-Manson power model for the acceleration factor in cyclic tests is only approximately valid;

3) when calculating the probability of failure-free operation of the module as a whole, we assume that during testing all legs are in the same conditions and fail with equal probability and independently;

4) as a result, we calculate the probability of failure of one leg and therefore assume that the ceramics is not subject to bending stresses and that it works purely on compression;

5) The presence of bending stresses in ceramics leads to the fact that the legs are in different conditions, and, therefore, the probabilities of their failure are different, as a result of which the presented theory ceases to be correct, which significantly complicates the calculations, since it requires determining the stresses in all the legs, and not just in one.

6) the traditional formula for the thermomechanical cracking stress of a layered material in the case of rigidly fastened thermoelectric leg is idealized.

The failure analysis showed first of all that despite the presence of significant shear thermomechanical stresses in the legs, no layer-by-layer cracking occurred. At the same time, bending stresses caused by the temperature gradient caused stress concentration in the most fragile parts of the soldered seams and solder failure. Long-term operation of the module in cyclic mode leads to the appearance of fatigue cracks, which gradually spread to the thermoelectric material. It was also found that moisture and material migration contribute to cracking to the greatest extent.

Thus, we see that the key means of increasing the cyclic stability of thermoelectric energy converters should be not only and not so much the search for hidden reserves in thermoelectric materials, but rather the leveling of thermomechanical stresses in thermoelectric legs due to design improvements of converters. Their developers quite often follow this path [7]. Incidentally, we note that in [1], the importance of damping thermomechanical stresses was emphasized in order to increase the cyclic stability of thermoelectric cooling modules, but specific requirements for damping elements of the design of thermoelectric energy converters were not determined.

Conclusions

1. By combining the strength of materials approach with the Weibull approach, a theory of damping of thermomechanical stresses in thermoelectric energy converters has been developed.

2. It has been established that the cyclic stability of thermoelectric energy converters increases significantly with increasing compliance of elastic damping elements. This increase is due to the large value of the Weibull shape parameter of the thermoelectric material.

3. It has been established that by properly selecting the rigidity coefficient, in particular the wire diameter and the diameter of the turns of the elastic damping elements in the form of cylindrical springs, it is possible to achieve complete compensation of the cracking stresses in thermoelectric legs caused by the temperature gradient, thereby dramatically increasing the cyclic stability of thermoelectric energy converters. As a result, there is no need to worry about the “safe” value of thermal conductivity from the point of view of thermomechanical cracking stresses, and, consequently, the thermoelectric figure of merit and efficiency of the thermoelectric material. An elastic element can be made, for example, in the form of a single-turn spring with an average diameter of 5 mm from aluminum wire with a diameter of no more than 3 mm. Since in practice a wire of a significantly smaller diameter is used, the lower limit of this diameter is determined not by the requirement for compensation of thermomechanical stresses, which is guaranteed to be performed with a significant reserve, but by its value, the minimum permissible from the point of view of the strength of connection.

4. The calculation results are in not only qualitative but also satisfactory quantitative agreement with the available experimental data.

5. The Coffin-Manson power model of the dependence of the acceleration factor in cyclic tests of thermoelectric energy converters on the temperature difference is substantiated. Its scope of applicability is the same as that of the Weibull approach, which relates the probability of maintaining the integrity of a thermoelectric leg to mechanical stresses therein. The Weibull parameters of a thermoelectric material are determined purely experimentally. The Coffin-Manson model is implemented in practice with acceptable accuracy.

References

1. Anatychuk L.I., Balaziuk V.M., Luste O.J., Malyshko V.V., Mikhalchenko V.P. (2003). On increasing the cyclic stability of thermoelectric cooling modules. *J. Thermoelectricity,* 4, 71 – 75.
2. Setty K., Sabbarayan G., Nguyen L. (2005). Power cycling reliability, failure analysis and acceleration factors of *Pb*-free solder joints. *Proceedings Electronic components and Technology Conference*, P. 907 – 915.
3. Pisarenko H.S., Kvitka O.L, Umanskyi E.S. (2004). *Strength of materials*: *manual*. Kyiv: Vyshcha shkola.
4. Sabo E.P. (2006). Mechanisms that determine the resource capabilities of thermoelectric converters. *J. Thermoelectricity,* 2, 59 – 70.
5. Karri N.K., Mo C. (2018). Reliable thermoelectric module design under opposing requirements from structural and thermoelectric considerations. *Journal of Electronic Materials,* 47, 3127 – 3135.
6. Park W., Barako M.T., Marconnet A.M., Asheghi M., Goodson K.E. (2012). Effect of thermal cycling on commercial thermoelectric modules. *13th IEEE ITHERM Conference*. (San Diego, CA, USA, 30 May 2012 – 01 June 2012).
7. Wereszczak A.A., Case E.D. (2015). *Mechanical response of thermoelectric materials.* Oak Ridge, TN: ORNL/TM-2015/227, U.S. Department of Energy.
8. *US Patent No 4011104* (1977). A. Basilius. Thermoelectric system.

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ДЕМПФУВАННЯ ТЕРМОМЕХАНІЧНИХ НАПРУЖЕНЬ ЯК ЗАСІБ ПІДВИЩЕННЯ ЦИКЛІЧНОЇ СТІЙКОСТІ ТЕРМОЕЛЕКТРИЧНИХ ПЕРЕТВОРЮВАЧІВ ЕНЕРГІЇ

На основі поєднання методів опору матеріалів з підходом Вейбулла, визначено вимоги до коефіцієнтів жорсткості демпфуючих елементів, які можна використати для зниження термомеханічних напружень у термоелектричних гілках з метою підвищення циклічної стійкості термоелектричних перетворювачів енергії. Обґрунтовано степеневу модель Коффіна-Менсона для залежності фактору прискорення за наявності циклічних температурних впливів від перепаду температури. Результати розрахунків знаходяться не лише у якісній, а й у задовільній кількісній згоді з експериментальними даними.

Ключові слова: циклічна стійкість, термоелектричний перетворювач енергії, термомеханічні напруження, демпфування, опір матеріалів, підхід Вейбулла, міцність на розтріскування, жорсткість пружного елемента.

Література

1. Анатичук Л.І., Балазюк В.М., Лусте О.Я., Малишко В.В., Михальченко В.П. Про підвищення циклічної стійкості термоелектричних модулів охолодження. // Термоелектрика. – 2003. – № 4. – С. 71 – 75.
2. Setty K., Sabbarayan G., Nguyen L. (2005). Power cycling reliability, failure analysis and acceleration factors of *Pb*-free solder joints. *Proceedings Electronic components and Technology Conference*, P. 907 – 915.
3. Писаренко Г.С., Квітка О.Л., Уманський Е.С. *Опір матеріалів*: підручник / за ред. Г.С. Писаренка. К.: Вища школа, 2004, 655 с.
4. Сабо Є.П. Механізми, що визначають ресурсні можливості термоелектричних перетворювачів. // Термоелектрика. – 2006. – № 2. – С. 59 – 70.
5. Karri N.K., Mo C. (2018). Reliable thermoelectric module design under opposing requirements from structural and thermoelectric considerations. *Journal of Electronic Materials,* 47, 3127 – 3135.
6. Park W., Barako M.T., Marconnet A.M., Asheghi M., Goodson K.E. (2012). Effect of thermal cycling on commercial thermoelectric modules. *13th IEEE ITHERM Conference*. (San Diego, CA, USA, 30 May 2012 – 01 June 2012).
7. Wereszczak A.A., Case E.D. (2015). *Mechanical response of thermoelectric materials.* Oak Ridge, TN: ORNL/TM-2015/227, U.S. Department of Energy.
8. *US Patent No 4011104* (1977). A. Basilius. Thermoelectric system.

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