DOI: 10.63527/1607-8829-2024-1-2-9-22

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FEATURES OF RENORMALIZATION OF THE ELECTRON SPECTRUM BY CONFINED PHONONS IN A SEMICONDUCTOR QUANTUM DOT-QUANTUM RING NANOSTRUCTURE

In the model of effective masses and rectangular potential energies for electron and the dielectric continuum model for phonons, a theory of renormalization of the electron energy spectrum by interaction with confined phonons in a semiconductor ($GaAs/Al_xGa_{1-x}As$) quantum dot-quantum ring nanostructure was constructed. The renormalized energy spectrum was found using the Green's function method by solving the Dyson equation. The dependences of partial and complete shifts of the ground electron energy level into the long-wavelength region of the spectrum on the geometric parameters of the semiconductor nanostructure were analyzed. The influence of shifts of the ground electron energy level on the efficiency of thermoelectric materials was considered. Bibl. 40, Figs. 2.
Key words: quantum dot, quantum ring, electron, phonon, energy spectrum, thermoelectric material.

Introduction

Currently, experimental possibilities for creating various semiconductor nanostructures allow scientists to grow entire ordered arrays of concentric single and double quantum rings with axial symmetry and study the luminescence spectra in them [1 – 3].

The unique properties of quasiparticles (electrons, holes, excitons, etc.) in such nanostructures, which manifest themselves during their interaction with each other and with external electric and magnetic fields, allow them to be used in modern nanoelectronics devices: semiconductor lasers [4], photodetectors [5] and elementary qubits of quantum computers [6].

Theoretical models for calculating spectra, wave functions of fundamental quasiparticles, and intensities of intraband and interband optical quantum transitions in such structures are also being intensively developed and improved.

In [7, 8], the authors investigated the dependence of electron energy spectrum in a simple cylindrical semiconductor quantum ring on the intensity of a uniform electric field directed perpendicular to the axial axis of the ring. They showed that these dependences for a certain range of changes in the electric field intensity are different and are determined by the ratio between the inner and outer radii of the rings. In particular, anticrossing of energy levels can be observed in the corresponding dependences.

In [9 – 15], the authors theoretically investigated the influence of electric and magnetic fields on the energy spectrum of an electron. In [16 – 19], the authors theoretically investigated the influence of

electric and magnetic fields on the energy spectrum, wave functions and intensities of intraband quantum transitions of an electron in double quantum nanorings based on $GaAs / Al_xGa_{1-x}As$ semiconductors. The stationary Schrödinger equation for a quasiparticle in the presence of a magnetic field is solved analytically exactly, and the wave function is obtained as a superposition of confluent hypergeometric functions and generalized Laguerre polynomials. In the case of an electric field, this equation is not solved exactly. Therefore, the spectrum of an electron interacting with an electric field was found by the method of expanding the unknown wave function of the electron by the complete orthonormal set of wave functions of a quasiparticle in a nanosystem without a field and solving the resulting secular equation. The authors showed that depending on the value of the magnetic field induction (B), the electric field intensity (F) and the ratio between the thicknesses of the nanorings, the electron in all states can be localized either in the inner or outer ring. In doing so, in the corresponding dependences of energy levels on F or B, anticrossings of energy levels are observed, and in the dependences of oscillator strengths, the maxima and minima of the intensities of the corresponding transitions are clearly expressed. The authors established that the reason for this behavior is a change in the localization of the electron in the space of two nanorings in different quantum states with a change in the electric field intensity or magnetic field induction.

The exciton and polaron effects in nanorings were studied in [20, 21]. Calculations performed by the authors showed that using electric and magnetic fields, it is possible to purposefully control the location of an electron in a system of double nanorings. The energy of polaron ground state decreases nonlinearly with increasing electric field intensity, and in general, the application of electric and magnetic fields leads to an enhancement of the electron-phonon interaction.

It is known that electron-phonon interaction is an important factor in determining the efficiency of thermoelectric materials [22 – 24]. Strong electron-phonon interaction can worsen the Seebeck coefficient due to increased electron scattering, but in certain materials it can contribute to improving the thermoelectric coefficient due to localized effects – quantum dots. Quantum dots are able to effectively scatter low-frequency phonons, which allows for optimal reduction of thermal conductivity without significant impact on electrical conductivity [25 – 28]. Therefore, optimization of this interaction by reducing thermal conductivity and maintaining high electrical conductivity is relevant, which is the key to creating highly efficient thermoelectric converters, including electricity sources and highly sensitive sensors [29 – 36].

In this work, the effect of confined phonons on the energy spectrum of an electron in a semiconductor nanostructure quantum dot – quantum ring will be theoretically investigated.

1. Theory of renormalization of the electron spectrum by interaction with confined phonons in a quantum dot–quantum ring nanostructure

The possibility of influencing the electron and phonon subsystems of new thermoelectric structures by another parameter – size – significantly expands the ways to improve the thermoelectric figure of merit. Two new directions are currently being considered. In the first direction, they are trying to achieve progress in the design and growth of superlattices, and in the second – to obtain ordered structures of lower dimensions (one-dimensional quantum wires and 0-dimensional quantum dots). The use of low-dimensional materials to increase thermoelectric efficiency has gained more opportunities due to the use of three concepts: energy throttling of carriers at barriers; "carrier-pocket" engineering; transition from semimetal to semiconductor. It should be noted that all these concepts and strategies are currently used to improve the efficiency of nanostructured thermoelectric materials in fundamental and

applied research in this area. Let us dwell on the analysis of the "carrier-rocket" technology [37]. Its essence lies in the design of the superlattice structure in such a way that one type of carrier is quantum confined in the quantum well region and another type of carrier of the same sign in the barrier region. This concept was introduced for the case of the *G*-point of electrons of *GaAs* quantum wells and for the *X*-point of electrons of *AlAs* barriers [38] in *GaAs*/AlAs quantum well superlattices [37].

Further, the paper investigates a simplified problem, a nanostructure consisting of a cylindrical semiconductor quantum dot (quantum well, *GaAs* medium), which is tunnel-coupled to a coaxial cylindrical nanoring (quantum well, *GaAs* medium) through a finite potential barrier ($Al_xGa_{1-x}As$ medium). The height of the nanostructure is *L*. The cross-section through the plane $z = 0$ and the diagram of potential electron energies of such a nanostructure are shown in Fig. 1.

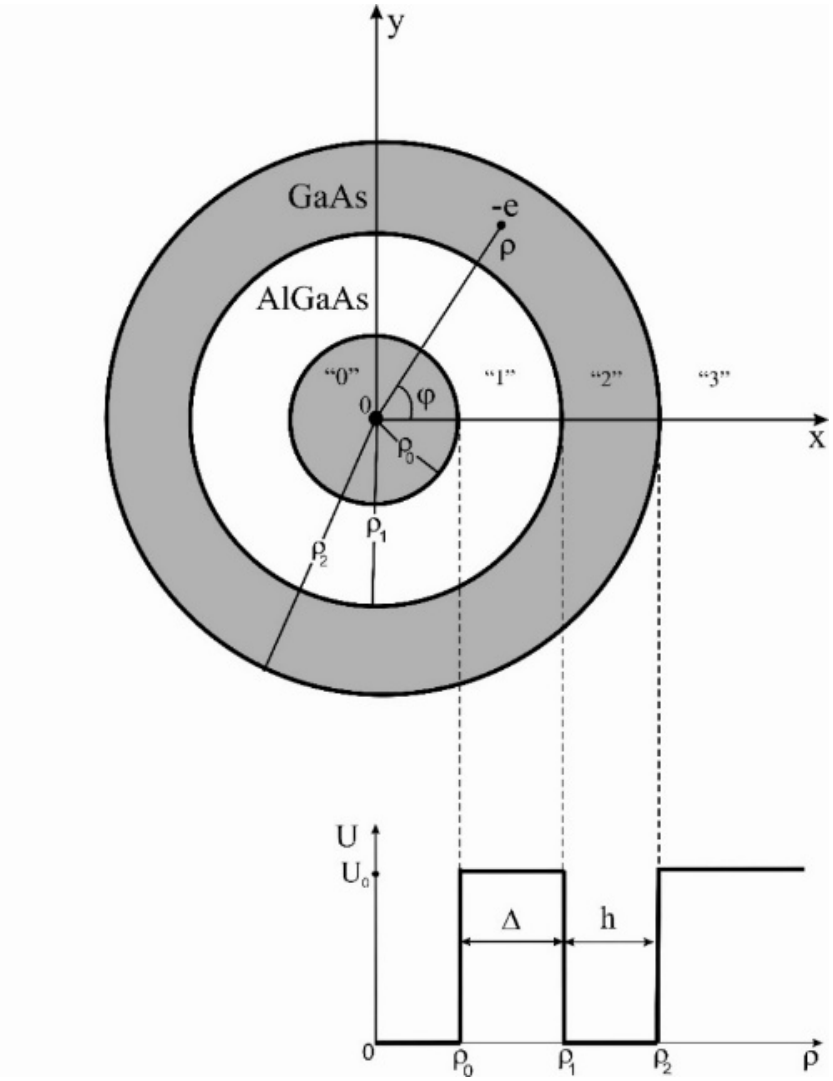

Fig. 1 Geometric and energy diagrams of the nanostructure.

For symmetry reasons, all further calculations are performed in a cylindrical coordinate system with the *Oz* axis along the axial axis of the nanostructure.

Since the lattice constants and dielectric constants of the semiconductor elements of the nanostructure differ little from each other, the model of effective masses and rectangular potential energies is used to calculate the spectra and wave functions of the electron, and the model of the dielectric continuum is used

to describe the phonon subsystem. Therefore, we will consider the effective masses, potential energies, and dielectric constants to be known in all regions of the nanostructure under study:

$$\mu^{(e)}(\vec{r}) = \begin{cases} \mu_0^{(e)}, & 0 \le \rho \le \rho_0, \quad \rho_1 \le \rho \le \rho_2 \\ \mu_1^{(e)}, & \rho_0 < \rho < \rho_1, \ \rho > \rho_2 \end{cases}, \tag{1}$$

$$U^{(e)}(\rho) = \begin{cases} U_0^{(e)}, & \rho_0 \le \rho \le \rho_1, \quad \rho > \rho_2, \\ 0, & 0 < \rho < \rho_0, \quad \rho_1 < \rho \le \rho_2. \end{cases}, \tag{2}$$

$$\varepsilon(\omega) = \begin{cases} \varepsilon_0(\omega), & 0 < \rho < \rho_0, \quad \rho_1 < \rho \le \rho_2, \\ \varepsilon_1(\omega), & \rho_0 \le \rho \le \rho_1, \quad \rho > \rho_2. \end{cases} \tag{2*}$$

The method for finding the energy spectrum ($E^e_{n_\rho m}$) and the electron wave functions ($\Psi_{n_\rho m}(\vec{r}_e)$ is discussed in detail in [39], so we will consider them known in the future. Here n_ρ and m are the radial and magnetic quantum numbers, respectively.

The polarization field potentials of confined phonons in the nanostructure under study are sought in the dielectric continuum model according to the general theory [40]. The expressions for them in the representation of second quantization over phonon variables are quite cumbersome, so we will not give them for now.

Now the complete Hamiltonian of the electron-phonon system looks like this:

$$\hat{H}(\vec{r}_e) = \hat{H}_e(\vec{r}_e) + \hat{H}_{e-L}(\vec{r}_e) + \hat{H}_L. \tag{3}$$

Here $\hat{H}_e(\vec{r}_e)$ is the Hamiltonian of the electron in the coordinate variables, $\hat{H}_L$ is the Hamiltonian of the confined phonons, already obtained in the representation of second quantization over its variables, $\hat{H}_{e-L}(\vec{r}_e)$ is the Hamiltonian of the interaction of the electron with confined phonons.

The transition to the representations of the filling numbers by electronic variables is carried out according to the general theory on quantized wave functions:

$$\begin{aligned} \hat{\Psi}_{n_\rho m}(\vec{r}_e) &= \sum_{n_\rho m k} \Psi_{n_\rho m}(\vec{r}_e)\, \hat{a}_{n_\rho m}, \\ \hat{\Psi}^+_{n_\rho m}(\vec{r}_e) &= \sum_{n_\rho m k} \Psi^*_{n_\rho m}(\vec{r}_e)\, \hat{a}^+_{n_\rho m}, \end{aligned} \tag{4}$$

where $a_{n_\rho m}, a^+_{n_\rho m}$ are the Fermi operators of annihilation and creation of electronic states.

Then the Hamiltonian of electrons and electron-phonon interaction in the representation of second quantization over all variables will look like

$$\hat{\mathbf{H}}_e = \int \Psi^+_{n_\rho m}\left(\hat{H}_e(\vec{r}_e) + \hat{H}_{e-L}(\vec{r}_e)\right) \Psi'_{n'_\rho m''} d^3\vec{r}_e = \hat{H}_e + \hat{H}_{e-L}. \tag{5}$$

Here

$$\hat{H}_e = \sum_{n_\rho m} E^e_{n_\rho m}\ a^+_{n_\rho m} a_{n_\rho m} \tag{6}$$

– electron Hamiltonian in the representation of second quantization in diagonal form

$$\widehat{H}_{e-L}=\sum_{p=0}^{3}\sum_{qs_p}\sum_{mm_1}\sum_{n_{\rho 1}n_{\rho 2}}F_{n_{\rho 1}m_1}^{n_{\rho 2}m_1+m}(m,q,k_{s_p})\times \\ \times\ \widehat{a}^{+}_{n_{\rho 1}m_1 q}\,\widehat{a}_{n_{\rho 2}m_1+m}\left(\widehat{b}^{+}_{mqs_p}+\widehat{b}_{-m-qs_p}\right) \tag{7}$$

– Hamiltonian of the interaction of an electron with confined L phonons with coupling functions

$$F_{n_{\rho 1}m_1}^{n_{\rho 2}m_1+m}(m,q,k_{s_0})=\sqrt{\frac{2e^2\Omega_{L0}}{L}\left(\frac{1}{\varepsilon_{\infty 0}}-\frac{1}{\varepsilon_{00}}\right)}\frac{1}{\rho_0\sqrt{k_{s_0}^2+q^2}}\frac{1}{\left|J_{m+1}(k_{s_0}\rho_0)\right|}\times \\ \times\int_0^{\rho_0}R^{(0)*}_{n_{\rho 1}m_1 q}(\rho)\,R^{(0)}_{n_{\rho 2}m_1+m}(\rho)\,J_m(k_{s_0}\rho)\,\rho\,d\rho \tag{8}$$

– when interacting with quantum dot phonons;

$$F_{n_{\rho 1}m_1}^{n_{\rho 2}m_1+m}(m,q,k_{s_1})= \\ =\sqrt{\frac{2e^2\Omega_{L1}}{L}\left(\frac{1}{\varepsilon_{\infty 1}}-\frac{1}{\varepsilon_{01}}\right)}\frac{\pi}{2}\frac{k_{s_1}}{\sqrt{k_{s_1}^2+q^2}}\times\left[\frac{1}{N_m^2\left(k_{s_1}\rho_1\right)}-\frac{1}{N_m^2\left(k_{s_1}\rho_0\right)}\right]^{-1/2}\times, \\ \times\int_{\rho_0}^{\rho_1}R^{(1)*}_{n_{\rho 1}m_1 q}(\rho)R^{(1)}_{n_{\rho 2}m_1+m}(\rho)\left[J_m\left(k_{s_1}\rho\right)-\frac{J_m\left(k_{s_1}\rho_0\right)}{N_m\left(k_{s_1}\rho_0\right)}N_m\left(k_1\rho\right)\right]\rho d\rho \tag{9}$$

– when interacting with phonons of the barrier layer;

$$F_{n_{\rho 1}m_1 k}^{n_{\rho 2}m_1+m}(m,q,k_{s_2})= \\ =\sqrt{\frac{2e^2\Omega_{L0}}{L}\left(\frac{1}{\varepsilon_{\infty 0}}-\frac{1}{\varepsilon_{00}}\right)}\frac{\pi}{2}\frac{k_{s_2}}{\sqrt{k_{s_2}^2+q^2}}\times\left[\frac{1}{N_m^2\left(k_{s_2}\rho_2\right)}-\frac{1}{N_m^2\left(k_{s_2}\rho_1\right)}\right]^{-1/2}\times, \\ \times\int_{\rho_1}^{\rho_2}R^{(2)*}_{n_{\rho 1}m_1 q}(\rho)R^{(2)}_{n_{\rho 2}m_1+m}(\rho)\left[J_m\left(k_{s_2}\rho\right)-\frac{J_m\left(k_{s_2}\rho_1\right)}{N_m\left(k_{s_2}\rho_1\right)}N_m\left(k_{s_2}\rho\right)\right]\rho d\rho \tag{10}$$

– when interacting with the phonons of the quantum ring;

$$F_{n_{\rho 1}m_1}^{n_{\rho 2}m_1+m}(m,q,k_3)=\sqrt{\frac{2e^2\Omega_{L1}}{L}\left(\frac{1}{\varepsilon_{\infty 1}}-\frac{1}{\varepsilon_{01}}\right)}\frac{1}{\rho_2\sqrt{k_{s_3}^2+q^2}}\frac{1}{\left|N_{m+1}(k_{s_3}\rho_2)\right|}\times \\ \times\int_{\rho_2}^{\infty}R^{(3)*}_{n_{\rho 1}m_1 q}(\rho)\,R^{(3)}_{n_{\rho 2}m_1+m}(\rho)\,N_m(k_{s_3}\rho)\,\rho\,d\rho \tag{11}$$

– when interacting with phonons of the external medium; J_m, N_m – Bessel functions of integer order ; $R_{n_\rho m}(\rho)$ – known radial electron functions.

Low-frequency (ε_0) and high-frequency (ε_∞) dielectric constants, as well as energies of confined phonons (Ω_L) are known and presented in Table 1.

The phonon-renormalized electron energies ($\tilde{E}^{(e)}_{n_\rho m}$) are determined from the dispersion equations

$$\omega - E^{(e)}_{n_\rho m} - M^{(e)}_{n_\rho m}(\omega) = 0\,, \tag{12}$$

where M is the mass operator related to the above coupling functions through the Green's function by the Dyson equation

$$M^{(e)}_{n_\rho m}(k,\omega) = \sum_{p=0}^{3} \sum_{s_p\, q\, m} \sum_{n_{\rho 1}\, m_1} \frac{\left| F^{n_{\rho 1}\, m_1 + m}_{n_\rho\, m}(m,q,k_{s_p}) \right|^2}{\omega - E^{(e)}_{n_{\rho 1}\, m_1}(q) - \Omega_{L_p}}\,. \tag{13}$$

For convenience, it is advisable to present these energies in terms of the energy $E^{(e)}_{n_\rho m}$ and the total shift of the corresponding electron energy level (Δ)

$$\tilde{E}^{(e)}_{n_\rho m} = E^{(e)}_{n_\rho m} + \Delta^{(e)}_{n_\rho m}\,. \tag{14}$$

The calculation and analysis of partial and corresponding full shifts of the ground electron level were performed by numerical methods for a nanostructure based on $GaAs$ / $Al_{0.4}Ga_{0.6}As$ semiconductors. The material parameters of the corresponding semiconductors are given in Table 1.

2. Analysis and discussion of results

Considering the weakness of the electron-phonon interaction, it is advisable to present the total shift of the ground electron level as the sum of the corresponding partial shifts.

Note that since the interaction of the electron with phonons is studied at $T = 0$, the ground level of quasiparticle is shifted only to the long-wavelength region, and the corresponding shifts are negative.

Table 1

$GaAs$	$Al_xGa_{1-x}As$
$\mu_0^{(e)} = 0.063 m_0$	$\mu_1^{(e)} = (0.063 + 0.083x) m_0$
$U_0^{(e)} = 0.57\left(1.155x + 0.37x^2\right)\ eV$	
$\varepsilon_{\infty 0} = 10.89$	$\varepsilon_{01} = 10.89 - 2.73x$
$\varepsilon_{\infty 0} = 12.9$	$\varepsilon_{01} = 12.9 - 2.84x$
$\Omega_{L0} = 35 meV$	$\Omega_{L1} = 35 + 1.83x + 17.12x^2 - 5.11x^3$

Fig. 2 shows the dependence of the partial shifts of the ground electron level due to the interaction with confined phonons of the quantum dot $\Delta^e_{L_0}$ (Fig. 2 *a*), the barrier layer $\Delta^e_{L_1}$ (Fig. 2 *b*), the quantum ring $\Delta^e_{L_2}$ (Fig. 2 *c*) and the corresponding total shift (Fig. 2 *d*) on the radius (ρ_0) of the quantum dot at $x = 0.4$, fixed thicknesses of the barrier layer $\Delta = 4a_{GaAs}$, the quantum ring $h = 4\ nm$. For convenience, the magnitudes of the shifts are given in dimensionless units relative to the energy of longitudinal optical phonons (Ω_{L_0}) of the $GaAs$ crystal.

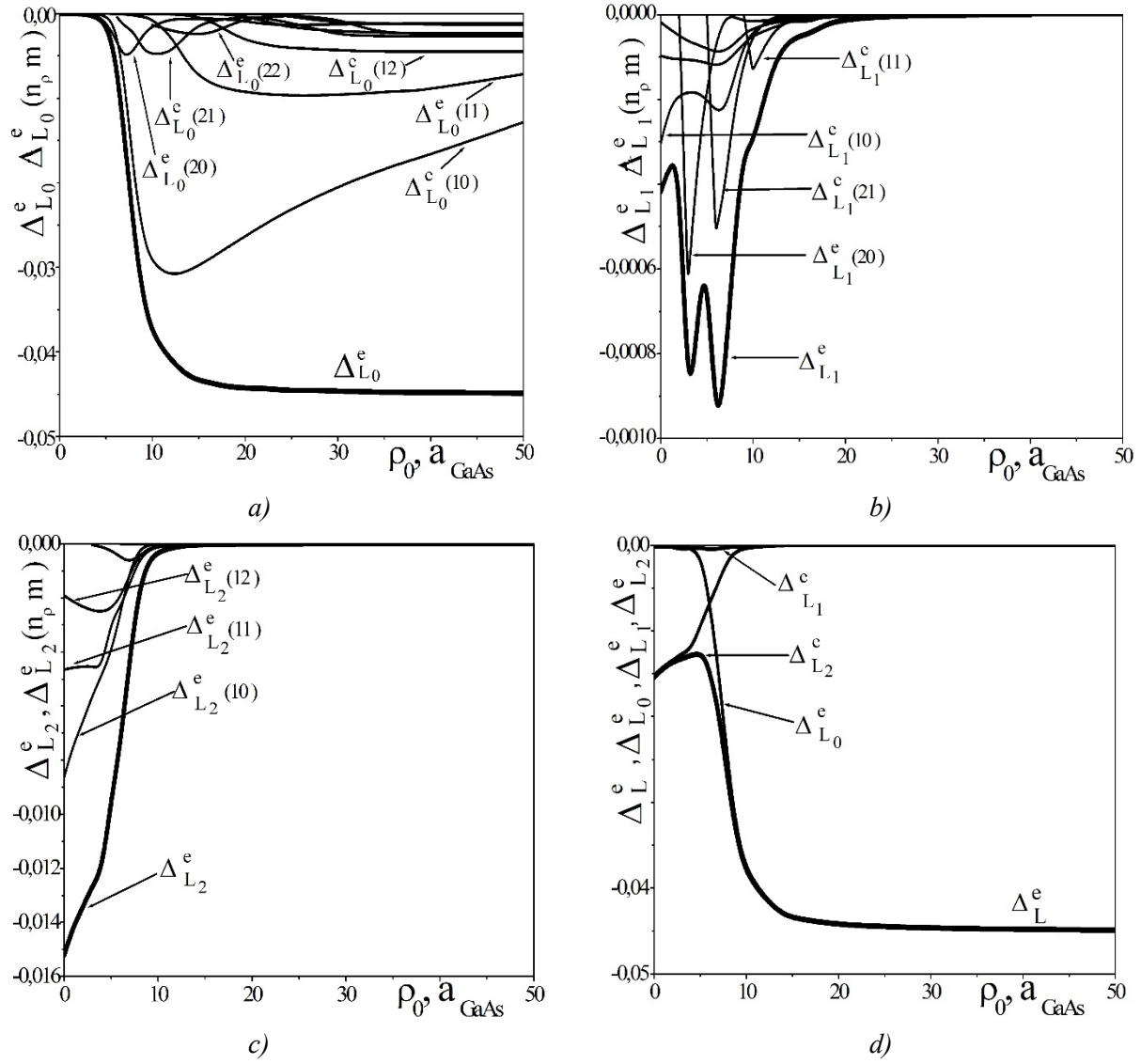

Fig. 2. Dependence of partial shifts of the ground electron level caused by the interaction with confined phonons of the quantum dot $\Delta^e_{L_0}$ (a), barrier layer $\Delta^e_{L_1}$ (b), quantum ring $\Delta^e_{L_2}$ (c) and the corresponding total shift Δ^e_L (d) on the radius (ρ_0) of the quantum dot at x = 0.4 and fixed thicknesses of the barrier layer $\Delta = 4a_{GaAs}$ and quantum ring h = 4 nm.

From Fig. 2 *a-c* it is seen that the shift of the ground electron level is formed by confined phonons of all media through intraband interaction ($\Delta^e_{L_0}(10), \Delta^e_{L_1}(10), \Delta^e_{L_2}(10)$) and kinematic interaction with energy levels with larger values of quantum numbers n_ρ and m through confined phonons. The contribution of intraband interaction with confined phonons of the quantum dot ($\Delta^e_{L_0}(10)$) to the absolute value $\Delta^e_{L_0}$ at small radii ρ_0 is also small, but slowly increases with increasing ρ_0. Starting from $\rho_0 \approx 5a_{GaAs}$, it first increases rapidly, reaching a maximum at $\rho_0 \approx 13a_{GaAs}$, and then slowly decreases (Fig. 2 *a*).

With increasing radius (ρ_0) of the quantum dot, more and more energy levels with different quantum numbers n_ρ and m appear in the nanostructure, the kinematic interaction with which forms the

corresponding partial contributions ($\Delta^e_{L_0}(n_\rho m)$). The behavior of these partial shifts depending on ρ_0 is similar to that caused by intraband interaction: after the appearance of the corresponding level, its contribution to the interaction first increases, reaching a maximum, and then slowly decreases. That is why the total shift of the ground level ($\Delta^e_{L_0}$), caused by the interaction with confined phonons of the quantum dot medium, smoothly increases with increasing ρ_0 with a slow tendency to saturation (bold curve in Fig. 2 *a*).

Similarly, the total shift ($\Delta^e_{L_2}$) is formed, caused by the interaction of the electron with confined phonons of the quantum ring medium (Fig. 2 *c*). However, unlike $\Delta^e_{L_0}$, $\Delta^e_{L_2}$ is significant at small radii (ρ_0) of the quantum dot, and with increasing ρ_0 it quickly decreases and, starting from $\rho_0 \approx 13a_{GaAs}$ is practically equal to zero.

As can be seen from Fig. 2 *b*, the shift of the ground electron level due to the interaction with the confined optical phonons of the barrier layer ($\Delta^e_{L_1}$) is orders of magnitude smaller than the corresponding shifts $\Delta^e_{L_0}$ and $\Delta^e_{L_2}$ and rapidly decreases with increasing ρ_0 .

Note that the total shift of the ground electron level due to the interaction with confined phonons of the external medium ($\Delta^e_{L_3}$) turns out to be an order of magnitude smaller than $\Delta^e_{L_1}$. That is why it is not shown in the figures, since its contribution to the renormalization of the ground electron state is negligibly small.

The described picture of the formation of partial shifts of the ground electron energy level due to its interaction with confined phonons is easy to understand from simple physical considerations. Indeed, at small radii of the quantum dot ($0 \le \rho_0 \le 5a_{GaAs}$), the electron is mainly located in the medium of the quantum ring, with a thickness h. As a result, it interacts most significantly with the L_2 phonons of this medium, forming a shift $\Delta^e_{L_2}$ (Fig. 2 *d*). With further growth of ρ_0, the electron increasingly penetrates into the medium of the quantum dot. In this case, naturally, its interaction with L_0 phonons increases, and with L_2 decreases. It is this circumstance that leads to the fact that, starting from $\rho_0 \approx 5a_{GaAs}$, the magnitude of the $\Delta^e_{L_0}$ shift sharply increases, and $\Delta^e_{L_2}$ decreases (Fig. 2 *d*). Starting from $\rho_0 \approx 13a_{GaAs}$, the electron is already completely localized in the quantum dot, and, accordingly, the total shift Δ^e_L is entirely determined by its partial shift $\Delta^e_{L_0}$ (Fig. 2 *d*). Shifting the ground electron level (Fermi level) in semiconductors can significantly affect their thermal conductivity, as it changes the concentration of charge carriers (electrons and holes), which play an important role in heat transfer. By reducing the thermal conductivity when shifting the ground electron level while keeping the electrical conductivity and thermoelectric coefficient practically unchanged, we will increase the figure of merit of new thermoelectric materials.

Note that for all values of ρ_0, the electron practically does not penetrate into the region of the barrier media ″1″ or ″3″. Therefore, it is not surprising that the contributions of L_1 - and L_3 - phonons ($\Delta^e_{L_1}$, $\Delta^e_{L_3}$) to the renormalization of the ground electron state are negligibly small.

Therefore, from the above analysis it is clear that the total shift of the ground electron level caused by the interaction with confined phonons (Δ^e_L) will be formed mainly by the interaction of the electron with the

corresponding phonons of the quantum dot medium ($\Delta^e_{L_0}$) and the quantum ring medium ($\Delta^e_{L_2}$) (Fig. 2 *d*). The contribution of confined phonons of the external medium ($\Delta^e_{L_3}$) and the barrier layer ($\Delta^e_{L_1}$) can be neglected for the geometric parameters of the multilayer nanostructure that are investigated in this work.

The developed theory of electron interaction with confined phonons in a multilayer quantum dot-quantum ring nanostructure allows optimizing thermoelectric structures by reducing thermal conductivity and maintaining high electrical conductivity, which is the key to creating highly efficient thermoelectric converters, including electricity sources and highly sensitive sensors.

Main results and conclusions

1. The theory of interaction of an electron with confined phonons in a multilayer quantum dot – quantum ring nanostructure has been developed using the Green's function method. The features of renormalization of the energy of the ground electron level by confined phonons depending on the radius (ρ_0) of the quantum dot have been investigated.
2. It has been established that the shift of the ground electron level is formed by confined phonons of all media through intraband interaction and kinematic interaction with the energy levels with larger values of quantum numbers n_ρ and m through confined phonons
3. It is shown that the full shift of the ground electron level caused by the interaction with confined phonons ($\Delta^{(e)}_L$) is formed mainly by the interaction of the electron with the corresponding phonons of the quantum dot medium ($\Delta^{(e)}_{L_0}$) and the quantum ring ($\Delta^{(e)}_{L_2}$). A decrease in thermal conductivity with a shift in the ground electron level with practically unchanged electrical conductivity and thermoelectric coefficient leads to an increase in the figure of merit of thermoelectric materials. The contribution of confined phonons of the external medium ($\Delta^{(e)}_{L_3}$) and the barrier layer ($\Delta^{(e)}_{L_1}$) for those geometric parameters of the multilayer nanostructure that are studied in this work can be neglected.

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Submitted: 15.02.2024.

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ОСОБЛИВОСТІ ПЕРЕНОРМУВАННЯ ЕЛЕКТРОННОГО СПЕКТРА ОБМЕЖЕНИМИ ФОНОНАМИ У НАПІВПРОВІДНИКОВІЙ НАНОСТРУКТУРІ КВАНТОВА ТОЧКА-КВАНТОВЕ КІЛЬЦЕ

У моделі ефективних мас і прямокутних потенціальних енергій для електрона та моделі діелектричного континууму для фононів побудовано теорію перенормування електронного енергетичного спектра взаємодією із обмеженими фононами у напівпровідниковій ($GaAs/Al_xGa_{1-x}As$) наноструктурі квантова точка – квантове кільце. Перенормований енергетичний спектр знаходився з використанням методу функцій Гріна шляхом розв'язування рівняння Дайсона. Проаналізовано залежності парціальних та повного зсувів основного енергетичного електронного рівня у довгохвильову область спектра від геометричних параметрів напівпровідникової наноструктури. Розглянуто вплив зсувів основного енергетичного електронного рівня на ефективність термоелектричних матеріалів. Бібл. 40, рис. 2.

Ключові слова: квантова точка, квантове кільце, електрон, фонон, енергетичний спектр, термоелектричний матеріал.

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Надійшла до редакції: 15.02.2024.