L. I. Anatychuk, *Academician of NAS of Ukraine*[1,2]
R. G. Cherkez, *dok. phys.– mat. sciences, professor*[1,2]
O. M. Porubanyi [2]
A. S. Zhukova [2]

[1]Institute of Thermoelectricity of the NAS and MES of Ukraine,
1 Nauky str., Chernivtsi, 58029, Ukraine,
[2]Yu.Fedkovych Chernivtsi National University, Chernivtsi, Ukraine

EFFECT OF LEG THICKNESS AND HEAT CARRIER VELOCITY ON THE EFFICIENCY OF A PERMEABLE GENERATOR THERMOELEMENT

The paper presents the results of computer research on the influence of leg thickness and gas pumping velocity for a 3D model of a permeable generator thermoelement on the EMF and efficiency. The dependences of the energy characteristics of a thermoelement made of materials based on Bi-Te-Se-Sb are calculated. Bibl. 11, Fig. 3, Table 1.

Key words: permeable thermoelement, computer simulation, electric energy generation, energy characteristics.

Introduction

The most widespread application of thermoelectric energy converters is based on the use of a thermocouple element [1, 2]. Its energy conversion efficiency is determined by the figure of merit parameters Z of the materials used. Therefore, the search for materials with the maximum value of the figure of merit becomes the main task of thermoelectric materials science. However, despite intensive research in this direction, no significant increase in the figure of merit has been observed over the past 20-30 years [3, 4]. The maximum values of the dimensionless figure of merit of thermoelectric materials for industrial use remain at the level of $1 \div 1.2$. Therefore, to improve efficiency, it is necessary to use new, non-traditional approaches, which consist in the use of other nonconventional variants of physical models of thermoelements which are the main component of a thermoelectric power converter.

One of them is the use of thermoelements with a developed internal heat exchange surface, namely permeable thermoelements. In such thermoelements, heat exchange with the heat source and heat sink occurs not only at the junctions, but also in the legs. Already the first theoretical [5] and experimental [6] studies for cooling gas flows have shown their promise. They indicate the possibility of improving the efficiency of energy conversion by a factor of $1.3 \div 1.4$.

However, such studies were conducted for a model that is difficult for practical implementation. Therefore, it is necessary to create and study a more realistic 3D model of a permeable thermoelement, which is the purpose of this paper.

Physical model and its mathematical description

The physical model of a permeable thermoelement, in which heat exchange with the heat carrier occurs not only through the connecting plates, but also with the lateral surfaces of the leg, is shown in Fig. 1. It includes *n* – and *p* – type legs (2, 10) covered by adiabatic insulation 1 and 8, which together form channels 5, 6, 9. Heat carrier 7 flowing through channels 3, 5 and 5, 9, is supplied through channel 6 with temperature T_h. The legs are made of a homogeneous material based on *Bi*-*Te* with a maximum value of figure of merit *Z* in the temperature range $20 \div 320\,^{\circ}$C. The temperature dependence of material parameters should be taken into consideration. Connecting plates *c* are made of copper, connecting resistance is 10^{-6} Ohm cm^2. The temperature T_0 of lower connecting plates is thermostated. The *n* – and *p* – type legs are interconnected by a thin layer 14, the thermal conductivity, electrical conductivity and thickness of which are neglected. The material of the legs is homogeneous and isotropic with known temperature dependences: electrical conductivity σ(*T*), the Seebeck coefficient α(*T*), thermal conductivity κ(*T*). In the thermoelectric medium, the volume effects of Thomson, Joule-Lenz and the Peltier contact effect are taken into consideration. The temperature of heat carrier at the inlet to the thermoelement was assumed to be equal to the temperature of hot junctions. The size of the thermoelement in the direction perpendicular to the plane of the figure is *d*, the value of *d* = *a*. The planes *d* = 0 and *d* = *a* are adiabatic insulations which form channels 5, 6, 9, Friction between the heat carrier and the adiabatic insulations 1, 8 is absent.

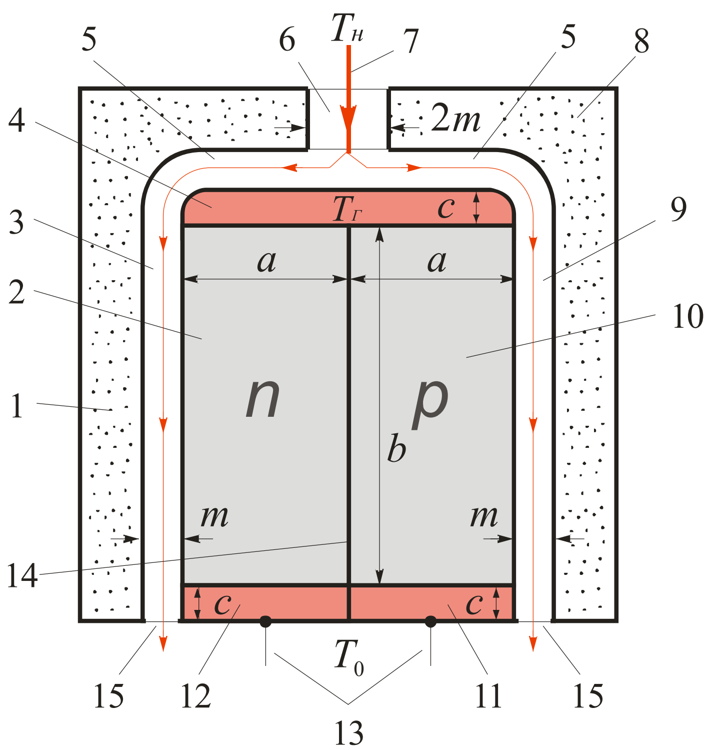


Fig. 1. Physical model of a permeable thermoelement:
1 – adiabatic insulation, 2 – n-type legs, 3 - channels, 4 – connecting plate, 5 - channel, 6 - channel, 7 – heat carrier, 8 – adiabatic insulation, 9 – channel, 10 – p-type leg, 11, 12 – connecting plates, 13 – electric contacts, 14 – layer between n- and p-type legs, 15 – channel outputs

On the lateral surface of legs 2 of connecting plates 4 which are in thermal contact with heat carrier 4, heat exchange is described by the Newton-Richmann law:

$$q_0 = \alpha_T (t - T), \tag{1}$$

where α_T is heat exchange coefficient, T is temperature of thermoelement leg, t is temperature of heat carrier.

The system of equations describing the distribution of temperature and potential in the thermoelectric medium is described by the fundamental laws of conservation of energy and current carriers [7]:

$$\vec{\nabla}\vec{W} = 0 \tag{2}$$

$$\vec{\nabla}\vec{i} = 0 \tag{3}$$

where $\vec{W} = \vec{q} + U\vec{i}$ is energy flux density.

Using the generalized Fourier and Ohm's laws for thermoelectric medium:

$$\vec{q} = -\kappa\vec{\nabla}T + \alpha\vec{i}\,T \tag{4}$$

$$\vec{i} = -\sigma(\vec{\nabla}U + \alpha\vec{\nabla}T) \tag{5}$$

where U is potential, κ is thermal conductivity, α is the Seebeck coefficient, σ is electric conductivity, one can obtain a system of differential equations to find the distribution of temperatures and potentials:

$$\left.\begin{aligned} &\vec{\nabla}\kappa\vec{\nabla}T + \frac{i^2}{\sigma} - T\vec{i}\vec{\nabla}\alpha = 0; \\ &\vec{\nabla}(-\sigma(\vec{\nabla}U + \alpha\vec{\nabla}T)) = 0. \end{aligned}\right\} \tag{6}$$

To describe the motion of the heat carrier in the channel, the system of the Navier-Stokes equations and the continuity equation are used, and for the distribution of heat carrier temperature the thermal conductivity equation is used.

The Navier-Stokes equations and the continuity equation can be written as [8]:

$$\left.\begin{aligned} &\rho\frac{d\vec{\vartheta}}{dt} = \rho\vec{F} - \vec{\nabla}P + \mu\vec{\nabla}^2\vec{\vartheta} + \frac{1}{3}\mu\vec{\nabla}(div\vec{\vartheta}), \\ &div\rho\vec{\vartheta} = 0. \end{aligned}\right\} \tag{7}$$

The left side (7) represents the fluid inertia force. The first term on the right side of (7) is the mass force, the second is the action of surface pressure forces (normal stresses), and the last two terms are the action of the contiguous components of surface forces (internal friction forces).

Heat exchange in the fluid is described by the thermal conductivity equation [9]:

$$\rho C_p\left(\frac{\partial T}{\partial t} + (\vec{\vartheta}\vec{\nabla})T\right) = -(\vec{\nabla}\vec{q}) + \sum_{i,j}\tau_{ij}S_{ij} - \frac{T}{\rho}\frac{\partial\rho}{\partial T}\bigg|_P\left(\frac{\partial\rho}{\partial t} + (\vec{\vartheta}\vec{\nabla})P\right) + Q \tag{8}$$

where ρ is the density, C_p is the heat capacity, T is the temperature, $\vec{\vartheta}$ – is the fluid velocity vector, q is the heat flux density, P is the pressure, τ_{ij} is the viscous stress tensor, where η is the viscosity, I is the unit tensor, $\vec{S}_{ij}$ is the strain rate tensor.

Since this problem is considered for the steady-state case, the left side of the first equation in system (7) is equal to zero. We also neglect the influence of mass forces, then the first term on the left side of the same equation is also equal to zero. Eq.(8) must also be written for the steady-state case, and heating of fluid due to internal friction, fluid compression, as well as heating of fluid due to internal heat sources are neglected. Then the system of Navier-Stokes, continuity and thermal conductivity equations for this problem will be written in the form:

$$\left.\begin{aligned} &-\vec{\nabla}P+\mu\vec{\nabla}^2\vec{\vartheta}+\frac{1}{3}\mu\vec{\nabla}(div\vec{\vartheta})=0,\\ &div\rho\vec{\vartheta}=0,\\ &\rho C_p(\vec{\vartheta}\vec{\nabla})T+\vec{\nabla}\vec{q}=0. \end{aligned}\right\} \quad (9)$$

Boundary conditions describing the conjugate problem used in this task have the following form:

– for thermoelectric medium:

$$\text{temperature - } \begin{matrix} T|_0=300K \\ T|_{z_h}=\alpha_T(T_h-T) \end{matrix}, \quad (10)$$

$$\text{potential - } \begin{matrix} U|_0=0 \\ U|_{x_3}=U_0 \end{matrix}, \quad (11)$$

– for heat carrier:

$$\text{velocity - } \begin{matrix} \vartheta|_0=\vartheta_0 \\ \vartheta|_{z_h}=P_0=0 \\ \vartheta|_{S_б}=0 \end{matrix}, \quad (12)$$

$$\text{inlet temperature - } t|_{z_h}=T_h \quad (13)$$

where ϑ_0 is the initial velocity of the heat carrier, U_0 is a fixed value of the potential, S_0 is the lateral surface of the thermoelement.

Implementation of the stated problem in the Comsol Multiphysics software package

To calculate the problem, the Comsol Multiphysics software package was chosen, namely, the equation in partial derivatives (*PDE modes)*. Where one of the ways to represent the equation is the coefficient form:

$$e_a \frac{\partial^2 \vec{u}}{\partial t^2} + d_a \frac{\partial \vec{u}}{\partial t} + \vec{\nabla}(-c\vec{\nabla}\vec{u} - \alpha\vec{u} + \gamma) + \beta\vec{\nabla}u + a\vec{u} = f \tag{14}$$

This equation is used for a thermoelectric medium and reduced to the form $\vec{\nabla}(-c\vec{\nabla}\vec{u}) = 0$. For this, e_a, d_a, α, γ, β, a are set to zero, and the value c is written in the form of a matrix:

$$c = \begin{pmatrix} \kappa + \alpha^2\sigma T + \sigma U\alpha & \alpha T\sigma + \sigma U \\ \alpha\sigma & \sigma \end{pmatrix}. \tag{15}$$

Moreover, vector $\vec{u}$ has also the form of a matrix:

$$\vec{u} = \begin{pmatrix} T \\ U \end{pmatrix}. \tag{16}$$

The Comsol Multiphysics – Non-Isothermal Flow module [11] is used to describe the motion and heat transfer of a fluid. The module includes the system of Navier-Stokes equations, the continuity equation and the heat transfer equation for a fluid that changes in time or in a steady-state mode. The calculation of the given model in the steady-state mode was carried out according to the relations:

$$\left.\begin{aligned} &\rho(\vec{\vartheta}\vec{\nabla})\vec{u}_2 = \vec{\nabla}\left[-PI + \mu(\vec{\nabla}\vec{\vartheta} + (\vec{\nabla}\vec{\vartheta})^T) - \frac{2}{3}\mu(\vec{\nabla}\vec{\vartheta})I\right], \\ &\vec{\nabla}(\rho\vec{\vartheta}) = 0, \\ &\rho C_p\vec{\vartheta}\vec{\nabla}T + \vec{\nabla}(\kappa\vec{\nabla}T) = 0. \end{aligned}\right\} \tag{17}$$

The value of the electric current was calculated through the integral over the cross-sectional area S_V:

$$I = \iint_{S_V} I_n dS_V , \tag{19}$$

where $I_n = n_x I_x + n_y I_y + n_z I_z$ is electric current density vector. The values I_x I_y I_z were determined by the relations:

$$I_x = -\sigma\frac{\partial U}{\partial x} - \sigma\alpha\frac{\partial T}{\partial x}, \tag{20}$$

$$I_y = -\sigma\frac{\partial U}{\partial y} - \sigma\alpha\frac{\partial T}{\partial y}, \tag{21}$$

$$I_z = -\sigma\frac{\partial U}{\partial z} - \sigma\alpha\frac{\partial T}{\partial z}. \tag{22}$$

The heat carrier flow rate was determined by integrating the velocity υ over the cross-sectional area of the channel S_{V1}:

$$G = \iint_{S_{V1}} \vartheta dS_{V1} \tag{23}$$

The electrical power of the thermoelement $W = I \cdot U$, heat flux coming to thermoelement $Q_h = GC_p \Delta t$.

The main parameter characterizing the efficiency of thermoelement work in the mode of electric energy generation is the efficiency which is determined by the relation:

$$\eta = \frac{W}{Q_h}. \tag{24}$$

Results of computer research on the characteristics of air permeable thermoelement made of materials based on *Bi-Te-Se -Sb*

The calculation was carried out for materials based on *Bi-Te-Se-Sb*. Functional dependences of the material parameters – the Seebeck coefficient α, thermal conductivity κ and electric conductivity σ on temperature were determined [3].

The simulation of a permeable thermoelement was made in the Comsol Multiphysics program for the following basic design (Fig. 1): height $b = 2$ *mm*, length 2 *mm*, width $a = 0.5$ *mm*. The dimensions of the lower interconnect – height $c = 0.1$ *mm*, length 2 *mm*, width $a = 0.5$ *mm*; upper – height $d = 0.1$ *mm*, length $c = 2$ *mm*, width 2 *mm*. The interconnect material is copper. These slots in the interconnect, together with the legs, form a system of channels for pumping the heat carrier. The contact resistance was $2 \cdot 10^{-6}$Ohm·cm^2. Fig. 2 shows the partition of such a construction into finite elements.

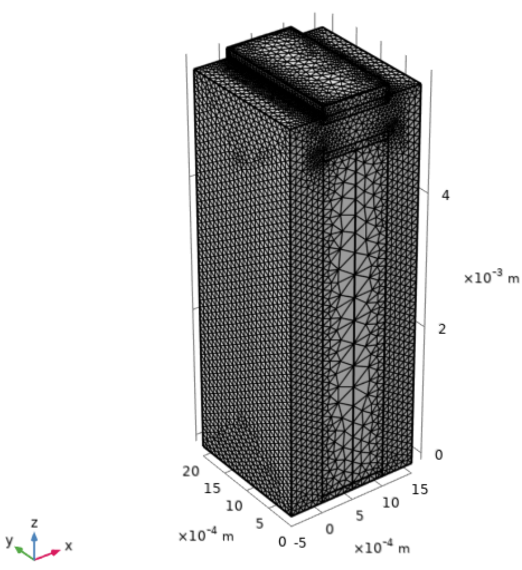
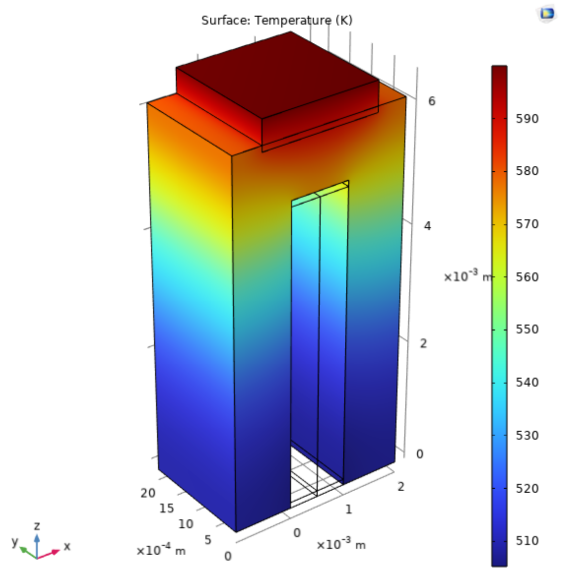
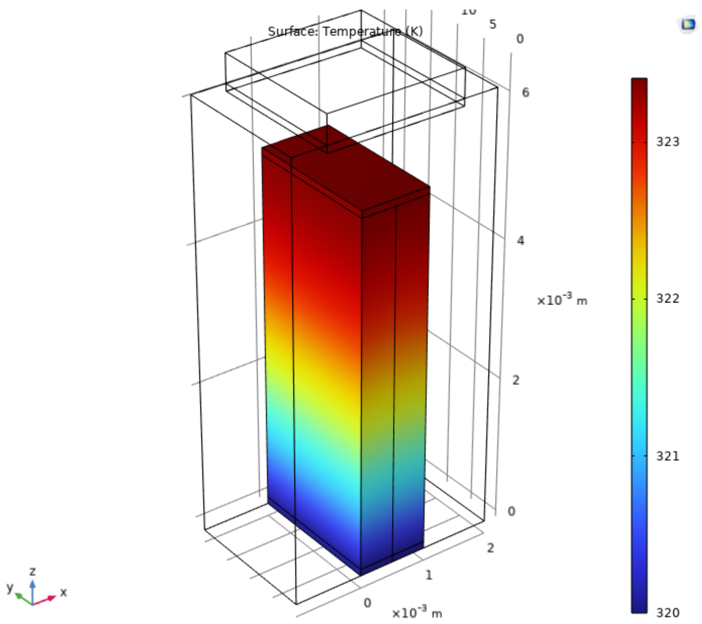


Fig. 2. Geometric grid.

An example of temperature distribution in heat carrier and thermoelement material at inlet gas temperature 600K is shown in Fig. 3.



a) temperature distribution in heat carrier;



b) temperature distribution in thermoelement material.
Fig. 3. Temperature distributions.

The width of thermoelement legs varied from 0.05 mm to 0.2 mm, and heat carrier velocity at the inlet to thermoelement varied from 0.001m/s to 0.05 m/s.The heat exchange coefficient - α_T, according to Newton-Richmann law, was 1000 $W/(m^2 \cdot K)$.

For the above parameters, the average integral characteristics of thermoelement were determined: air temperature at the thermoelement outlet - t_{out}; thermoelement electromotive force - EMF; the value of the electric current *I*, *A*; air consumption - *G*; electric power *W*; efficiency η. The dependences of these parameters on leg width *a* and height 0.002m for different heat carrier velocities *V* at the thermoelement inlet are presented in Table. The heat carrier temperature at the thermoelement inlet was 600K.

Table

Dependences of energy parameters on the width of leg and the velocity of heat carrier

a, ***m***	***V,*** ***m/s***	$t_{вих}$**,** **K**	***EPC,*** **V**	***I,*** **A**	***G,*** ***m³/s***	***W*, 10⁻⁸** **W**	η **(%)**
0.002	0.001	506.80	4.09E-04	1.25E-05	0.000855	0.50929	1.52
0.002	0.01	518.18	4.18E-04	1.32E-05	0.008724	0.551344	2.19
0.002	0.03	543.75	4.47E-04	1.49E-05	0.027447	0.665821	2.18
0.002	0.05	558.25	4.62E-04	1.59E-05	0.046937	0.732695	2.18
0.001	0.001	506.75	6.81E-04	6.71E-06	8.55E-04	4.57042	2.05
0.001	0.01	523.25	7.17E-04	7.31E-06	0.008823	5.24203	1.64
0.001	0.03	471.62	5.71E-04	4.14E-06	0.022122	2.3663	0.54
0.0005	0.001	474.75	0.001043	2.11E-06	7.75E-04	2.19625	0.11
0.0001	0.001	511.87	0.005279	9.16E-08	8.67E-04	4.83785	0.177

As can be seen from the table, for a leg width of 0.002 m, the value of efficiency from the rate of heat carrier supply to the thermoelement reaches saturation with a small maximum at a rate of 0.01 m/s. With a decrease in the thickness of legs (0.001 m), we obtain lower efficiency values at the same heat carrier velocities. Therefore, to identify maximum efficiency, it is necessary to carry out multi-parameter optimization of a permeable thermoelement. It is difficult to solve such a problem using the Comsol Multiphysics software package by parameter selection, since for modern computers with a clock frequency of 4.7 GHz, the program searches for a solution to a program with one set of parameters for 8 hours. Therefore, to solve a multi-parameter optimization problem, it is more expedient to use the mathematical theory of optimal control developed for a 1D model of a permeable thermoelement.

Conclusions

1. A 3D model of a permeable generator thermoelement was developed in the Comsol Multiphysics software package.
2. The temperature distributions in the material of thermoelement legs and heat carrier, potentials in the thermoelement, air velocities and energy characteristics of a permeable generator thermoelement made of materials based on *Bi-Te-Se-Sb* were determined.
3. For multi-parameter optimization of structural and thermophysical parameters of a permeable thermoelement, it is advisable to use the mathematical theory of optimal control developed for a 1D model of a permeable thermoelement.

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Анатичук Л. І., *акад. НАН України*[1,2]
Черкез Р. Г., *док. фіз.-мат. наук, професор*[1,2]
Порубаний О. М. [2]
Жукова А. С. [2]

[1]Чернівецький національний університет
імені Юрія Федьковича, вул. Коцюбинського 2,
Чернівці, 58012, Україна
[2]Чернівецький національний університет
імені Юрія Федьковича, вул. Коцюбинського 2,
Чернівці, 58012, Україна

ВПЛИВ ТОВЩИНИ ВІТКИ ТА ШВИДКОСТІ ТЕПЛОНОСІЯ НА ЕФЕКТИВНІСТЬ ПРОНИКНОГО ГЕНЕРАТОРНОГО ТЕРМОЕЛЕМЕНТА

В роботі представлені результати комп'ютерних досліджень по впливу товщини вітки та швидкості прокачки газу для 3D моделі проникного генераторного термоелемента на ЕРС та ККД. Розраховано залежності енергетичних характеристик термоелемента з матеріалів на основі Bi-Te-Se-Sb. Бібл. 11, рис.3, табл. 1.

Ключові слова: проникний термоелемент, комп'ютерне моделювання, генерація електричної енергії, енергетичні характеристики.

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