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Computer-Aided Study of a Thermoelement with Developed Lateral Heat Exchange

The work is devoted to the study of thermoelectric generator elements with developed lateral heat exchange, which allows increasing their efficiency. Using the Comsol Multiphysics software environment, a three-dimensional model of a thermoelement with developed lateral heat exchange was created. The influence of the leg height and the velocity on the efficiency, the power generated by the thermoelement, the voltage and other characteristics was studied.

Key words: computer design, permeable structures, thermoelectric generator elements, efficiency, electric power.

Introduction

The development of thermoelectric technologies plays a key role in increasing the efficiency of energy conversion systems, especially in conditions of autonomous energy supply and waste heat recovery [1–4]. One of the promising areas for improving thermoelectric devices is design optimization of thermoelements which ensures an increase in their energy efficiency. In particular, an important factor is the use of heat transfer on the lateral surface of thermoelement [5, 6], which can significantly affect the temperature distribution and operating characteristics of the device.

In [7–9] it is shown that changing the geometry of legs and thermal conditions on the lateral surfaces of the thermoelement makes it possible to control heat exchange processes, which in turn affects the efficiency of thermoelectric energy conversion [10-13]. In [14],

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methods for calculating the optimal parameters of two models of a thermoelectric converter in the mode of electrical energy generation are described, namely a sectional converter with the movement of the coolant along the heat-absorbing junctions of thermoelements and a converter of permeable thermoelements, in which the coolant passes through channels located along the height of the thermoelement legs. The comparison of efficiency showed that permeable thermoelements can outperform traditional ones and reach sectional ones. Therefore, it is relevant to develop work [15] on optimizing the design of such thermoelements in a 3D model to achieve their maximum efficiency. The results of the analysis will allow us to assess the effectiveness of various design solutions and formulate recommendations for optimizing thermal elements with lateral heat exchange for generating electrical energy using waste heat from vehicles and other thermal waste.

The purpose of this work is to create a computer 3D model of a thermoelement with developed lateral heat exchange and analyze its efficiency depending on the height of the legs and the coolant velocity.

1. Physical model of a permeable thermoelement in electric energy generation mode

The physical model of a permeable thermoelement is shown in Fig. 1. The physical model includes *n*- and *p*-type legs 2 and 10, respectively, and electrical connecting plates 4, 11, 12. The legs and connecting plates are covered by adiabatic insulation 1 and 8, which together form channels 5, 6, 9. Through channel 6, a coolant 7 with a temperature of T_H,is supplied, which flows through the channels 3, 5, 9 and exchanges heat with the connecting plates and legs of the thermoelement. The temperature T_0 of the lower connecting plates is fixed. The legs of *n*- and *p*-type are interconnected by a thin electrically insulating layer 14.

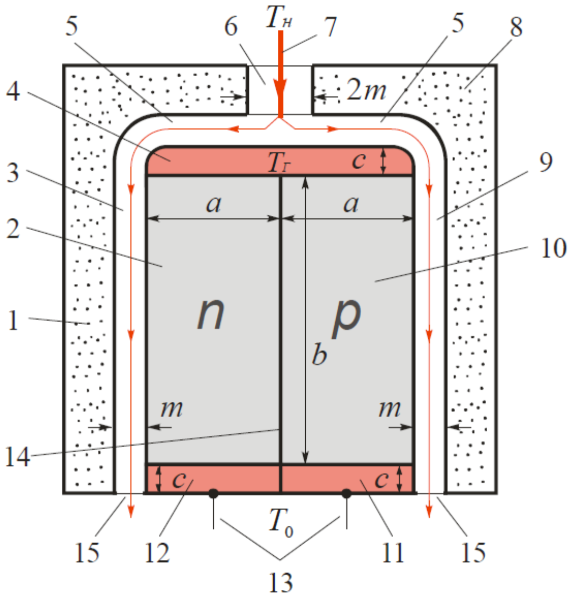

Fig. 1. Physical model of a permeable thermoelement: 1, 8 – adiabatic isolation, 2, 10 – n- and p-type legs, 3, 5, 6, 9 –channels, 4, 11, 12 – connecting plates, 7 – coolant, 13 – electric contacts, 14 – layer between n-and p-type legs, 15 – channel outputs

The size of the thermoelement in the direction perpendicular to the plane of the figure is *d*, the value of d = 2a. The planes *d* = 0 and *d* = *a* form the front and rear walls of channels 3, 5, 6, 9. They are adiabatically isolated. In the model, there is no friction between the coolant and the walls of the channels.

The legs are made of a homogeneous material based on Bi-Te with a maximum value of figure of merit Z in the temperature range of 20÷320°C. The material of the legs is homogeneous and isotropic with known temperature dependences of the electrical conductivity σ(T), thermoEMF α(T) and thermal conductivity κ(T). The Thomson, Joule-Lenz and Peltier bulk effects are taken into account in the thermoelectric medium. The model also takes into account the electrical contact resistance between the connecting plates and the thermocouple legs. The electrical connecting plates are made of copper.

2. Mathematical description and implementation of the model in COMSOL Multiphysics

The COMSOL [16] Multiphysics simulation environment provides a variety of tools and capabilities for studying physical phenomena and designing models used in scientific research. To implement the described model, we used the Multiphysics environment module, which includes the following modules.

Module *Laminar Flow*

The module interface is used to calculate velocity and pressure fields for a homogeneous fluid flow in laminar flow regime.

The equations solved using the Laminar Flow interface are the Navier-Stokes equation for conservation of momentum (1) and the continuity equation for conservation of mass (2):

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\nabla\mathbf{v}\right)=-\nabla p+\mu\nabla^2\mathbf{v}+\mathbf{f}\,, \tag{1}$$

$$\frac{\partial \rho}{\partial t}+\nabla(\rho\mathbf{v})=0\,, \tag{2}$$

where ρ is the density of the coolant, **v** is the fluid velocity vector, *t* is time**,** p is pressure, μ is the viscosity coefficient, f is the external forces.

Let us consider the stationary case in the absence of external forces. We assume that the coolant is incompressible, and we neglect internal friction. Then equations (1), (2) will take the following form:

$$\rho\left(\mathbf{v}\nabla\mathbf{v}\right)=-\nabla p\,, \tag{3}$$

$$\rho\nabla\mathbf{v}=0\,, \tag{4}$$

For the channel walls, we apply a boundary condition for the velocity that excludes the sliding of the coolant

$$\mathbf{v}=0\,, \tag{5}$$

at the inlet, coolant velocity

$$\mathbf{v}=-v_0\mathbf{n}\,, \tag{6}$$

where **n** is the normal vector to the coolant inlet plane, v_0 is the linear velocity of the coolant at the inlet.

At the channel exits, we assume that the coolant does not experience resistance. The boundary conditions will be

$$p = 0 .$$

Module *Heat Transfer in Fluids*

The interface of this module is used to simulate heat transfer in fluids by thermal conduction, convection, and radiation.

The heat transfer equation in the general case has the form:

$$\rho C_p \left(\frac{\partial T}{\partial t} + \mathbf{v} \nabla T \right) = \nabla \left(\kappa \nabla T \right) + Q , \qquad (7)$$

where C_p is heat capacity at constant pressure, Q is the heat source.

In our case (stationary, without heat sources) equation (7) will have the form

$$\rho C_p \mathbf{v} \nabla T = \nabla \left(\kappa \nabla T \right). \qquad (8)$$

The boundary condition at the coolant inlet is

$$T = T_H , \qquad (9)$$

At the outlet, the coolant transfers heat only through mass transfer:

$$\mathbf{q} = -\nabla \left(\kappa \nabla T \right) = 0 . \qquad (10)$$

At the boundaries of the channels that are in contact with the thermoelectric legs, the heat exchange conditions according to the Newton-Richmann law are fulfilled.

$$-\mathbf{n} \cdot \mathbf{q} = h \left(T_{TE} - T \right) \qquad (11)$$

where h is heat transfer coefficient, T_{TE} is thermoelement leg temperature.

All other boundaries are adiabatically isolated

$$-\mathbf{n} \cdot \mathbf{q} = 0 \qquad (12)$$

The modules described above allow you to find velocity and temperature fields in the coolant.

Let us now consider the phenomena occurring in the legs of the thermoelement. For this purpose, the following modules were used.

Module *Heat Transfer in Solids*

In this module, the equation for finding the temperature distribution T corresponds to the differential form of Fourier's law:

$$\rho C_p \frac{\partial T}{\partial t} = \nabla (\kappa \nabla T) + Q_J , \qquad (13)$$

where Q_{ext} is heat source. For the stationary case, Fourier's law has the form

$$-\nabla (\kappa \nabla T) = Q_J . \qquad (14)$$

where Q_J is the Joule heat

$$Q_J = \mathbf{j} \nabla U , \qquad (15)$$

j is electric current density (19.)

The boundary conditions for the thermoelement legs are:
– the cold side temperature of the thermoelement

$$T = T_C, \tag{16}$$

– heat flow through the boundaries of the thermoelement in contact with the coolant

$$-\mathbf{n}\cdot\mathbf{q} = h\left(T_{HC} - T\right), \tag{17}$$

where T_{HC} is coolant temperature.

All other boundaries of thermoelement legs are adiabatically isolated.

Module *Current Conservation*

This module uses the law of conservation of electric current to calculate the electric potential field *U.* In the stationary case, it has the following form:

$$\nabla\mathbf{j} = 0, \tag{18}$$

where **j** is electric current density. It is determined by the generalized Ohm's law

$$\mathbf{j} = -\sigma\nabla U - \sigma\alpha\nabla T, \tag{19}$$

where the second term on the right-hand side is responsible for thermoelectric phenomena.

The boundary conditions for the task are as follows: One of the electric contacts is *Ground*:

$$U = 0. \tag{20}$$

Another electric contact depending on operating mode of thermoelement is either electrically isolated (EMF mode)

$$\mathbf{nj} = 0, \tag{21}$$

or meets the following condition (optimal load mode):

$$U = U_0 = \varepsilon / 2, \tag{22}$$

where ε is electromotive force of thermoelement obtained in EMF mode.

Other thermoelement boundaries are electrically isolated (21).

All the above modules operate simultaneously in an iterative calculation cycle to calculate the interconnected fields of velocity v, temperature *T* and electric potential *U* in the corresponding elements of thermoelement design.

The efficiency of a thermoelement is determined by the formula

$$\eta = \frac{W}{Q_H}, \tag{23}$$

where *W* is electrical power of thermoelement, Q_H is thermal power supplied with the coolant.

$$W = IU_0. \tag{24}$$

electric current flowing through the thermoelement is determined by integration over the area of one of the electrical contacts:

$$I = \iint_S \mathbf{j}d\mathbf{S}. \tag{25}$$

where $d\mathbf{S}$ is elementary surface vector (normal to the surface).

The thermal power supplied to the thermoelement is determined by integration over the area of the input channel:

$$Q_H = \iint_S \left(-k\nabla T + \rho C_p \mathbf{v}(T - T_0)\right) d\mathbf{S}, \tag{25}$$

where T_0 is ambient temperature.

Studies of this kind were conducted for a permeable thermocouple in a 3D model, and the results were first obtained in [15]. The influence of the coolant purge rate and the supply voltage of the thermocouple on the temperature difference and energy conversion efficiency were investigated. Here, we will determine the optimal value of the coolant velocity and determine the optimal height of the thermoelement legs with developed lateral heat exchange in the electric energy generation mode.

3. Simulation results

The thermoelement was simulated for the following geometric configuration (Fig. 1): height $b = 10$ mm, $d = 1$ mm, width $a = 0.5$ mm. The thickness of connecting plates $c = 0.1$ mm, width of channels 3, 9 m =0.1 mm, width of channel 6 – 2m.

Temperature dependences of kinetic coefficients of thermoelectric material based on Bi-Te, which were used when simulating, were obtained at the Institute of Thermoelectricity [17]. The material of the connecting plates is copper. The coolant is air.

The variable parameters of the simulation were the leg height d and the linear velocity of the coolant v at the inlet (Fig. 2 *a*, inflow). The leg height varied in the range of 1 – 15 mm. The characteristics of the thermoelement were studied for the coolant velocity in the range of 0.1 – 3 m/s. The coolant temperature at the inlet $T_H = 600$ K, the cold side temperature $T_0 = 300$ K.

Fig. 2 shows a computer representation of a thermoelement model with developed lateral heat exchange. For high accuracy of calculations, a finite element mesh with high discretization was chosen. In this model, it contained ~ 10^6 elements.

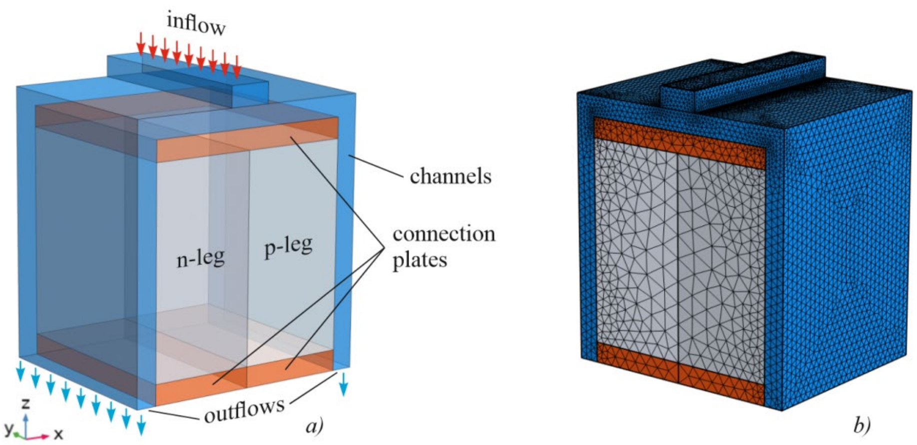

Fig. 2. Computer model of a thermoelement: a) thermoelement design, b) finite element mesh

Fig. 3, 4 show examples of calculated velocity and temperature fields of the coolant in the channels of the thermoelement. The figures show a section of the plane *zx* for a leg height of $d = 1$ mm and a coolant velocity at the inlet of v = 3 m/s.

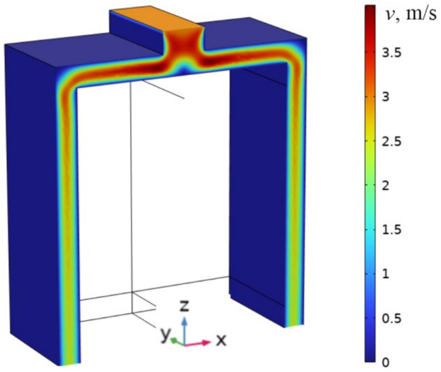

Fig. 3. Coolant velocity field in the thermoelement channels

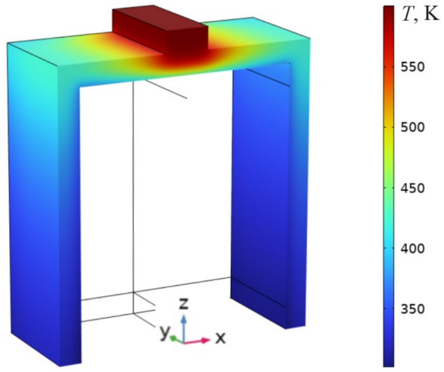

Fig. 4. Coolant temperature field in the thermoelement channels

As it passes through the channels, the coolant exchanges heat with the thermoelement legs, creating a temperature difference in them, due to which an electromotive force (EMF) is generated in the thermoelement.

Fig. 5 shows an example of temperature distribution in the thermoelement legs; Fig. 6 shows an example of electric potential distribution in them. The difference in electric potentials on the contacts is the generated EMF.

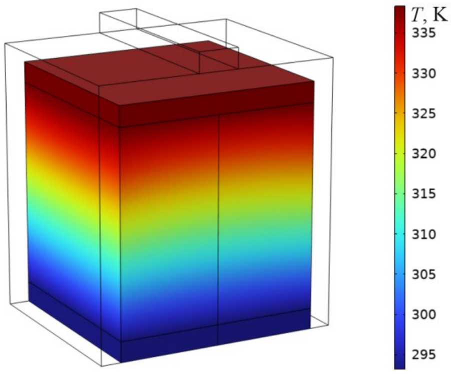

Fig. 5. Temperature distribution in thermoelement legs

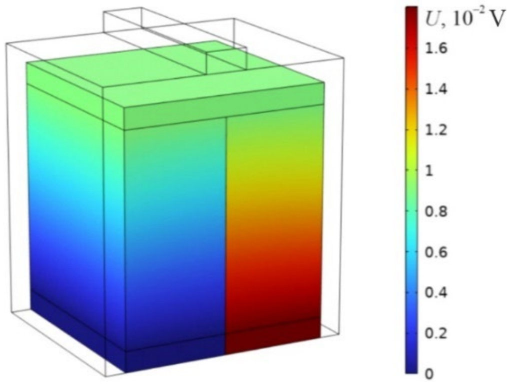

Fig. 6. Electric potential distribution in thermoelement legs

Fig. 7 shows plots of thermoelement efficiency with an optimal electrical load versus the inlet velocity of the coolant and the height of the legs.

The simulation of a thermoelectric generator for different lengths of the leg (1 mm, 5 mm, 10 mm, 15 mm) showed that the coolant velocity significantly affects its energy characteristics. It was determined that when the velocity increases to 0.8 – 1 m/s, the electromotive force of the thermoelement, as well as the voltage and electric current on the load, reach saturation, following which a further increase in velocity does not provide a significant increase in efficiency.

The optimal range of velocities of 0.8 – 1 m/s is observed for all investigated lengths of the thermoelement, which indicates a general pattern in the processes of heat transfer and electricity generation. High velocities (~ 3 m/s) do not provide a significant improvement in performance, whereas at too low velocities a significant drop in efficiency is observed. This confirms that the heat transfer process in the thermoelement channel has an efficiency limit due to both the intrinsic properties of the material and the thermal contact with the coolant flow.

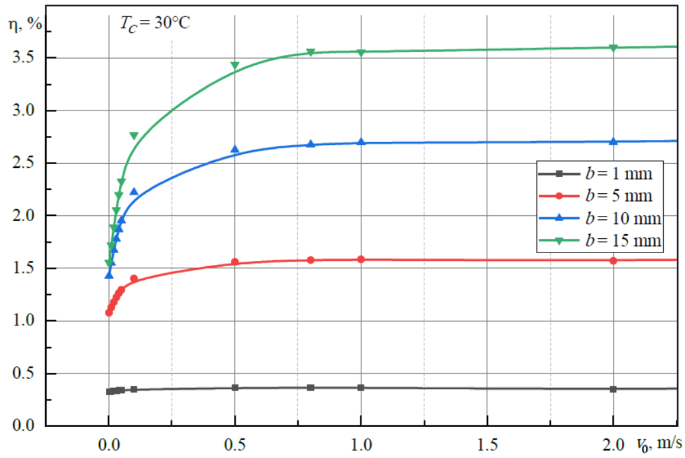

Fig. 7. Dependence of thermoelement efficiency on coolant inlet velocity for different heights of thermoelement legs

Fig. 8 shows the plots of electric power of a thermoelement with optimal electric load versus coolant inlet velocity and legs height.

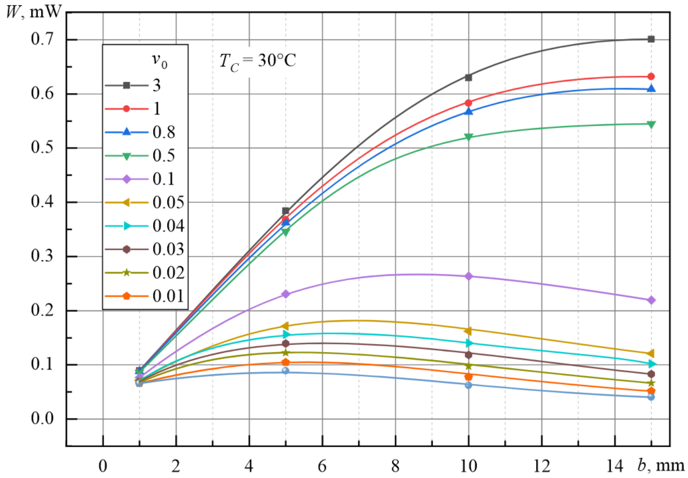

Fig. 8. Thermoelement electric power versus coolant inlet velocity for different heights of thermoelement legs

As can be seen from Fig. 7, 8, further increase in the height of the leg does not provide a significant improvement in efficiency, and the power reaches saturation or even decreases (for low coolant velocities). In addition, increasing the height leads to an increase in the cost of semiconductor material, its mass and the complexity of manufacturing the thermoelement.

The results obtained indicate the need to take into account the features of thermal contact [17], the dependence of heat transfer in the channels on the air velocity, channel diameter, and temperature conditions when designing thermoelectric elements with developed lateral heat transfer. The specified velocity range allows achieving the maximum efficiency, which is a key factor for increasing the energy efficiency of such generators in practical operating conditions [18-23]. Functional gradient and segmental thermoelements are promising for increasing the efficiency of useful action [24-27]. The use of lateral heat exchange in them can create situations in which the efficiency of energy conversion will be additionally increased. The technologies for obtaining such thermoelements are similar to the technologies for creating modules [28-30] for various purposes.

Conclusions

1. The rational coolant velocity for thermoelectric generators is within 0.8 – 1 m/s and provides the best ratio between generation efficiency and coolant purge costs.
2. Increasing the velocity above 1 m/s does not significantly increase efficiency, whereas velocities that are too low significantly reduce the electrical power generated by the thermoelement.
3. Increasing the height of thermoelectric legs to 15 mm allows you to get more electrical power from the thermoelement at coolant velocities above 0.5 m/s. Further increase in the height of the leg is impractical due to the increase in cost and decrease in the reliability of such a thermoelement.

The obtained simulation results can be used to optimize the design of a generator thermoelement with developed lateral heat exchange.

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Комп'ютерне дослідження термоелемента з розвиненим бічним теплообміном

Робота присвячена дослідженню термоелектричних генераторних елементів із розвиненим бічним теплообміном, що дозволяє підвищити їхню ефективність. Використовуючи програмне середовище Comsol Multiphysics, було створено тривимірну модель термоелемента з розвиненим бічним теплообміном. Досліджено вплив висоти вітки та швидкості подачі теплоносія на коефіцієнт корисної дії (ККД), потужність що генерується термоелементом, напругу та інші характеристики.

Ключові слова: комп'ютерне проектування, проникні структури, термоелектричні генераторні елементи, ефективність, електрична потужність.

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