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OPTIMAL CONTROL OF TRANSIENT THERMOELECTRIC COOLING PROCESS IN THE MODE OF MINIMUM POWER CONSUMPTION

The problem of optimal control of transient thermoelectric cooling process in the mode of minimum power consumption is formulated and a method for its solution is proposed. An algorithm and a computer tool have been developed, which are used to calculate the optimal time dependences of the thermoelement supply current, whereby a given cooling temperature is reached within a given time with minimum power consumption. Examples of computer simulation of such optimal control functions for transient cooling process are given. It has been established that energy saving when supplying thermoelements with an optimally time-dependent current reaches 25 - 50% in comparison with the option of direct current power supply.Bibl. 29, Fig.5, Tabl. 1.

Key words: transient thermoelectric cooling, optimal control, optimal time dependences of thermoelement supply current.

Introduction

Thermoelectric cooling is used in various spheres of human life. But the process of thermoelectric cooling has been studied in detail and optimized mainly for steady-state operating modes of a thermoelectric converter. At the same time, back in the 50s of the 20th century, in the theoretical work of L.S. Stilbans and N.A. Fedorovich [1] it was shown that in transient modes it is possible to achieve deeper cooling than in steady-state ones. This fact was later confirmed by many theoretical and experimental studies [2 – 9] and continues to be intensively studied by modern researchers [10 – 18].

The sphere of practical application of transient modes of thermoelectric coolers concerns cases when object cooling time is of vital importance. These, for example, can be coolers for laser devices, to improve the image quality in night vision devices, thermal imagers and other military devices, as well as to quickly remove heat pulses that are released during the operation of electronic components [19, 20].

It is possible to achieve the advantages of transient cooling modes over steady-state ones if these modes are optimized. The most rational problems of optimization of the process of transient thermoelectric cooling are associated with the search for optimal control functions of this process, in particular, the optimal time dependences of the supply current of thermoelements. For the first time, such theoretical problems were considered in [6, 7, 21, 22] for the simplest models of thermoelectric converter, which did not take into account such important factors as the influence of the Thomson effect, contact resistance at thermoelement junctions, heat release and heat capacity of the cooled object, and the like. Approximate analytical methods for solving such problems were proposed and, accordingly, approximate results were obtained.

Modern computer methods of searching for optimal time functions of current for thermoelectric coolers in transient modes mainly consist in choosing the best function from a limited set of given time dependences, rather than in solving optimization problems [23, 24].

The problems of transient cooling process optimization are related to the problems of optimal control of objects with distributed parameters [25]. These are complex problems for which there are no generalized solving methods. Therefore, the development of methods for solving the problems of optimal control of the dynamic modes of thermoelectric coolers is an urgent task.

In [26 – 28], approaches were proposed to solve the problem of finding the optimal time dependence of the current to achieve the lowest cooling temperature for a given time. The purpose of this work is to develop, on the basis of optimal control theory, methods and an algorithm for optimizing the process of transient thermoelectric cooling in the mode of minimum power consumption, to calculate the optimal time dependences of thermoelement supply current, whereby the specified cooling temperature is reached with minimum power consumption, and to analyze the efficiency of using such current functions.

Formulation of the problem of optimal control of transient cooling process in the mode of minimum power consumption

The thermoelement model used to optimize transient cooling is shown in Fig. 1. The following important physical factors and rational approximations are taken into account in the mathematical description of the model.

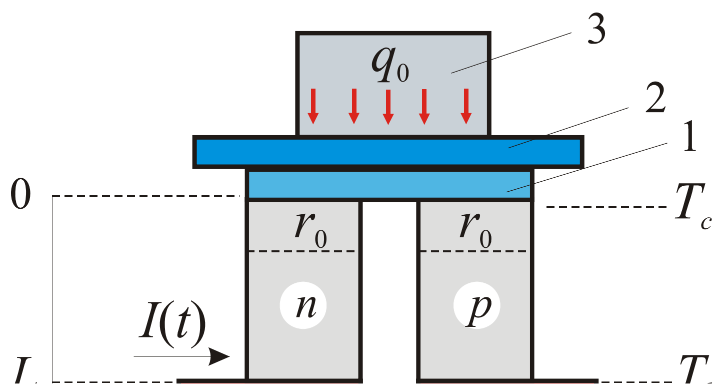


Fig. 1. Model of thermoelement for optimization of transient cooling process.
1 – connecting plate, 2 – insulating plate, 3 – cooled object.

1. The temperature distribution in n- and p-type legs is considered to be one-dimensional, the temperature depends on coordinate x along the height of the legs and changes with time.

2. The material of the legs is homogeneous, the thermoelectric properties – specific heat c, resistivity ρ, the Seebeck coefficient α, thermal conductivity κ can approximately be considered to be temperature-independent and identical for legs of both conduction types.

3. The Thomson heat absorption is taken into account in the bulk of the legs. The Thomson coefficient β can approximately be considered to be a constant value.

4. The Joule heat release on the contact resistance that takes place in the zone of contact with connecting plates on the cold junction of thermoelement legs is taken into account.

5. The process of transient cooling is essentially affected by the heat capacity and heat release of the cooled object, the heat capacity of the insulating and connecting plates, as well as the heat exchange

between the cold surface of module with the environment The cooled object together with the insulating and connecting plate is considered to be a cumulative object with a concentrated heat capacity, the temperature of which is equal to the temperature of the cold junction of the thermoelement and depends on time. Heat exchange of the cold surface of the module with the environment of constant temperature occurs according to Newton's law.

6. The temperature of the hot surface of the module is considered to be fixed.

Under these assumptions, the thermal processes in both thermoelement legs are similar and described by transient thermal conductivity equation in the form

$$c\frac{\partial T}{\partial t}=\kappa\frac{\partial^2 T}{\partial x^2}+\rho\frac{I^2(t)}{s^2}-\beta\frac{I(t)}{s}\frac{\partial T}{\partial x} \quad , \tag{1}$$

where $T(t,x)$ is temperature, $I(t)$ is current is the leg which in the general case is a function of time. Time t changes on the interval $t\in[0,t_1]$, and coordinate x is directed along the leg from the cold to the hot junction (Fig.1) and changes on the section $x\in[0,L]$, L is the height, s is the cross-section of the leg.

In this equation, the first term on the right side takes into account the thermal conductivity in the thermoelement leg, the second term – the Joule heat release, the third – the Thomson heat.

The initial condition of the problem of transient thermoelectric cooling is generally given by

$$T(x,0)=T_a, \tag{2}$$

where T_a is ambient temperature.

The boundary conditions take into account the heat balance on the heat-absorbing surface and temperature stabilization of the heat-releasing thermoelement and are written as follows:

$$g\frac{\partial T(0,t)}{\partial t}=\left[q_0+Ks\left(T_a-T\right)-\alpha I(t)T+I^2(t)\frac{r_0}{s}+\kappa S\frac{\partial T}{\partial x}\right]_{x=0} \tag{3}$$

$$T(L,t)=T_h, \tag{4}$$

In condition (3), g is a total volumetric heat capacity of the cooled object, the insulating and connecting plates, calculated for one thermoelement leg. The first term on the right side of (3) takes into account heat release q_0 of the cooled object, the second – heat exchange between the surface and the environment, K is heat exchange coefficient, the third – the Peltier heat, the fourth- the Joule heat release due to contact resistance the value of which is r_0.

The process of transient thermoelectric cooling can be controlled by changing the current I over time within $I\in G_I$, $G_I=\{I_{\min}, I_{\max}\}$. One of the rational optimization problems is to determine the optimal current function $I(t)$ such that for a certain period of time t_1 will assure a given object cooling temperature T_c under the condition of maximum coefficient of performance (COP) of transient process.

By definition, the COP is determined by the relation: $\mathrm{COP}=\dfrac{Q_L}{J}$, where Q_L is thermal load $Q_L=g(T_a-T_c)\equiv\mathrm{const}$, which for the formulated problem is a given value, J is energy consumption. Therefore, the maximum COP corresponds to the minimum energy consumption.

Thus, it is necessary to find the optimal control function $I(t)$ such that satisfies the condition of reaching at a final time moment t_1 of a given temperature T_c,:

$$T\left(0,t_1\right)=T_c\,, \tag{5}$$

and assures minimum power consumption which is determined by the functional

$$J=\int_0^{t_1}\left[\alpha I(t)(T_h-T(0,t))+(\rho+\frac{r_0}{L})\frac{L}{s}I^2(t)\right]dt\,. \tag{6}$$

This problem refers to the problems of optimization of an object with distributed parameters [25], the behavior of which is described by a boundary value problem in parabolic equations (1) - (3).

An efficient way to solve problems of optimization of an object with distributed parameters is its discretization along the coordinate and thus obtaining an object with lumped parameters, which is described by a system of ordinary differential equations [26]. This makes it possible to use the Pontryagin maximum principle [29] for optimization.

Method of solving the problem. Optimality conditions

$T_0(t)$:

$$T_0(t)=\int_0^{t}\left[\alpha I(t)(T_h-T(0,t))+(\rho+\frac{r_0}{L})\frac{L}{s}I^2(t)\right]dt\,. \tag{7}$$

We pass in equations (1) - (4) to the dimensionless coordinate x = x / L, and discretize them by the coordinate. This procedure allows us to write equations (1), (3) as a system of ordinary differential equations, and the boundary value problem (1) - (4) is written as follows:

$$\frac{\partial T_i}{\partial t}=f_i(T(t),I(t)),\quad i=0,1,...N\,, \tag{8}$$

$$T_{N+1}\left(t\right)=T_h\,, \tag{9}$$

where N=1/h is the number of nodes in the coordinate, h is a step in the coordinate, and functions f_i are given by:

$$f_0(t)=\alpha I(t)(T_h-T_1(t))+(\rho+\frac{r_0}{L})\frac{L}{s}I^2(t)\,,$$

$$f_1(t)=\frac{1}{g}\left[q_0+Ks\left(T_a-T_1(t)\right)-\alpha I(t)T_1(t)+I^2(t)\frac{r_0}{s}+\kappa\frac{s}{L}\frac{T_2(t)-T_1(t)}{h}\right], \tag{10}$$

$$f_i(t)=\frac{\kappa}{cL^2h^2}(T_{i+1}(t)-2T_i(t)+T_{i-1}(t))+\rho\frac{I^2\left(t\right)}{cs^2},\quad i=2,...N\,.$$

The initial conditions for the discretized system (8) are as follows:

$$T_0\left(0\right)=0,\quad T_i\left(0\right)=T_a,\quad i=1,...N\,, \tag{11}$$

Condition (5) and functional J (6) take on the form

$$T_1(t_1) = T_c\,, \tag{12}$$

$$J = T_0(t_1). \tag{13}$$

The problem is to find such a function $I(t)$ and the corresponding solution $T_i(t)$ i=0,1, N of the system of equations (8) with the initial conditions (11), whereby condition (12) is satisfied for a certain moment of time t1 and for functional (13) takes its minimum value.

This problem refers to the problems of optimal control of objects, which are described by the equations of motion for phase variables T under the given values of some functions from phase variables at the finite moment of time t_1.

Such a given function is condition (12) which is written as

$$F(T_1(t_1)) \equiv T_c - T_1(t_1) = 0\,, \tag{14}$$

and instead of functional J (13) we consider an expanded functional

$$\Phi = J + \nu F\,, \tag{15}$$

where ν is an unknown parameter that must be chosen so as to satisfy condition (12).

Then the formulated optimization problem becomes a problem for phase variables with a free right end and a fixed time, the solution of which is given by the Pontryagin maximum principle [29].

To solve the problem, the Hamiltonian function is written according to the rule

$$H = \sum_{i=0}^{N} \psi_i f_i(T, I, t)\,, \tag{16}$$

where the unknown functions (pulses) ψ_i are solutions of the auxiliary system of equations

$$\frac{d\psi_i}{dt} = -\frac{\partial H}{\partial T_i}, \quad i = 0,...,N \tag{17}$$

with conditions at point $t = t_1$ (transversality conditions) in the form

$$\psi_i(t_1) = -\frac{\partial \Phi(T(t_1))}{\partial T_i}, \quad i = 0,...,N\,. \tag{18}$$

Optimal control function $I_{opt}(t)$ is found from the Pontryagin maximum condition

$$H(T(t), I_{opt}(t), \psi(t), t) = \max_{I \in G_I} H(T(t), I, \psi(t), t)\,, \tag{19}$$

that is, function $H(T(t), I(t), \psi(t), t)$ of variable I at each $t\in[0,t_1]$ reaches a maximum at point $I = I_{opt}(t)$ for all $I\in G_I$.

For our formulated problem the Hamiltonian function (16) acquires the form

$$H = -f_0(T_1, I, t) + \sum_{i=1}^{N} \psi_i f_i(T, I, t), \tag{20}$$

and the system of equations (17) with transversality conditions (18) for pulses ψ is written as

$$\begin{aligned}
&\frac{d\psi_1}{dt} = -\psi_0 \frac{\partial f_0}{\partial T_1} - \psi_1 \frac{\partial f_1}{\partial T_1} - \psi_2 \frac{\partial f_2}{\partial T_1}, \\
&\frac{d\psi_2}{dt} = -\psi_1 \frac{\partial f_1}{\partial T_2} - \psi_2 \frac{\partial f_2}{\partial T_2} - \psi_3 \frac{\partial f_3}{\partial T_2}, \\
&\frac{d\psi_i}{dt} = -\psi_{i-1} \frac{\partial f_{i-1}}{\partial T_i} - \psi_i \frac{\partial f_i}{\partial T_i} - \psi_{i+1} \frac{\partial f_{i+1}}{\partial T_i}, \quad i = 3,...,N-1, \\
&\frac{d\psi_N}{dt} = -\psi_{N-1} \frac{\partial f_{N-1}}{\partial T_N} - \psi_N \frac{\partial f_N}{\partial T_N}.
\end{aligned} \tag{21}$$

$$\psi_1(t_1) = \nu, \qquad \psi_i(t) = 0, \qquad i = 2,...,N. \tag{22}$$

Thus, the condition for the maximum (19) of the Hamiltonian function H (20) in conjunction with the basic system of ordinary differential equations (8) with the initial conditions (11) and the associated auxiliary system (21) with conditions at the final moment of time (22), which depend on the parameter ν, and which should provide the specified cooling temperature T_c at the final time moment t_1, set the solution to the problem of optimizing the process of transient thermoelectric cooling in the mode of minimum power consumption. Optimality conditions (19) - (22) make it possible to determine the optimal current function $I_{opt}(t)$ for such a mode.

Obviously, the complexity of such an optimization problem allows it to be solved only by computer methods. To solve it, based on the method of successive approximations, an algorithm was developed and a software tool was created in the MathLab environment.

Results of optimization of transient cooling process

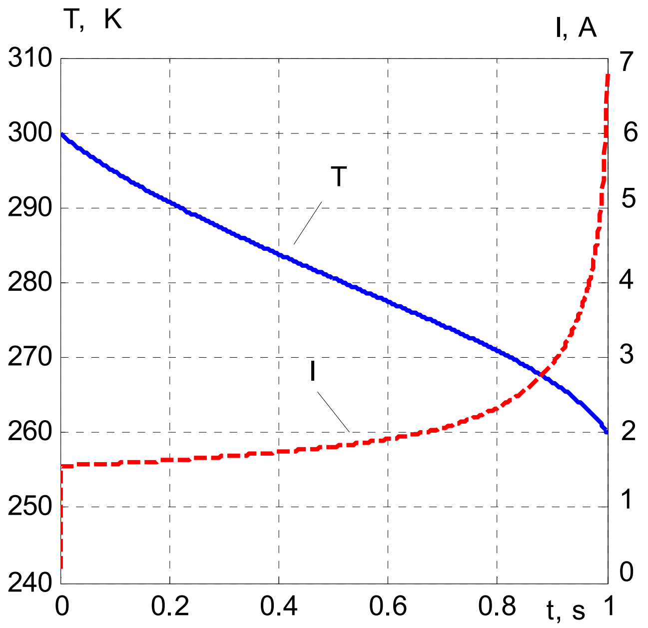
Calculations of optimal current functions $I_{opt}(t)$ and characteristics of transient cooling process were carried out by the example of a thermoelement whose legs are made of Bi_2Te_3 based materials. Used for this purpose, the values of material parameters and other values that characterize transient cooling process are listed in Table.

Table

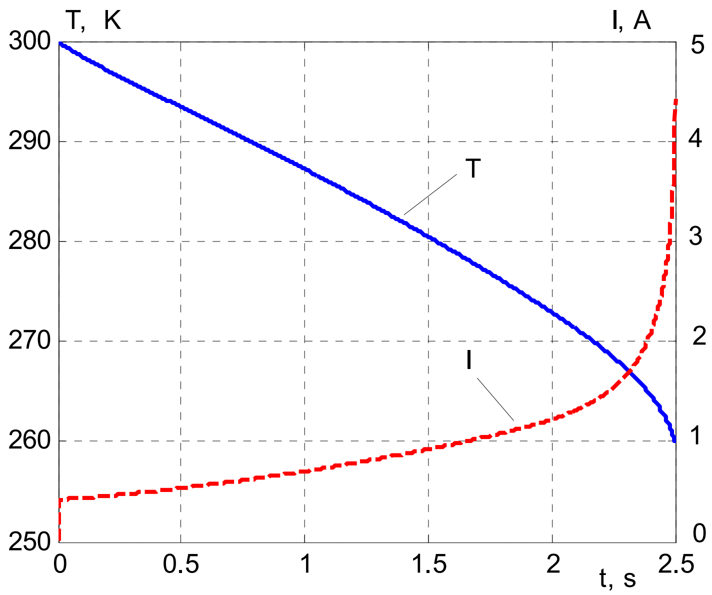
Parameter values used for calculations

Parameter	Value
Specific heat c, J/cm^3·K	1.4
Seebeck coefficient α, μV/K	200
Resistivity ρ, Ohm·cm	10^{-3}
Thermal conductivity κ, W/cm·K	0.015
Thomson coefficient β, μV/K	75
Contact resistance r_0, Ohm·cm^2	$5\cdot10^{-6}$
Total volumetric heat capacity g, J/ K	$1.25\cdot10^{-3}$
Heat release q_0, W	0.001
Heat exchange coefficient K, W/ cm^2·K	0.001
Ambient temperature T_a, K	300
Leg height L, cm	0.14
Leg cross-section s, cm^2	0.01

Optimal functions $I_{opt}(t)$ were calculated for different time intervals of reaching given cooling temperature under condition of minimum power consumption. Examples of such functions calculated for temperature reduction from 300 K to 260 K for 1 s and for 2.5 s are given in Fig. 2. It is obvious that for different time intervals these functions are different. Fig. 2 also shows the reduction of temperature with time to reach its given value under conditions of using these optimal dependences of current.



a)



б)

Fig. 2. Optimal functions of current and their respective functions of temperature reduction from 300 K to 260 K for 1 s (a) and 2.5 s (b).

Fig. 3 illustrates the temperature distribution which is set in thermoelement legs for 1 s and 2.5 s.

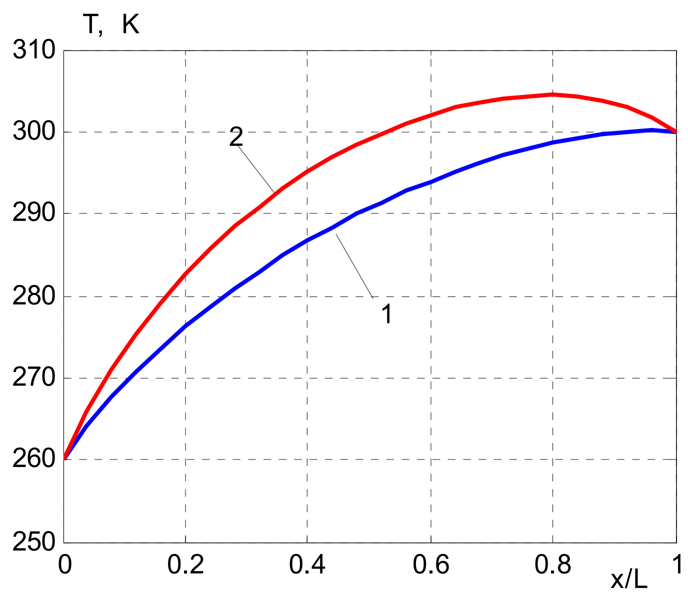


Fig. 3. Temperature distribution in thermoelement legs for 1 s (2) and 2.5 s (1).

Fig. 4 shows the results of calculating the dependence of the power consumed by the thermoelement on the time of reaching the cooling temperature of 260 K under the conditions of application of the optimal current functions. Power consumption significantly depends on the time interval during which the set temperature must be reached. Increasing the time interval leads to a decrease in power consumption. There is an optimal time interval during which cooling to a given temperature is achieved with the lowest power consumption. The results in Fig. 4 show that the specified cooling temperature under the conditions of supplying the thermoelement with a current that varies in time according to the optimal dependence can be achieved both in a short period of time with a certain power consumption, and for a longer time, but with a significantly lower, namely 2 - 2.5 times, power consumption.

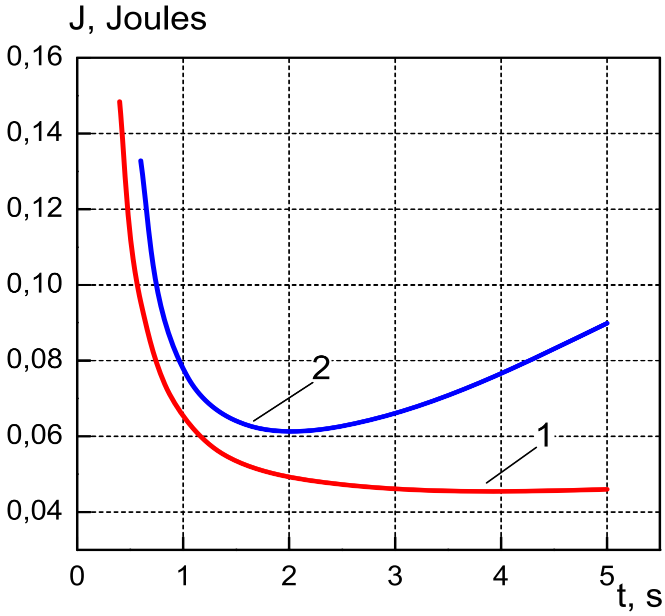


Fig. 4. Time dependences of thermoelement power consumption under the conditions of using optimal functions of current (1) and direct current (2).

Also, the calculations of power consumption were carried out when the thermoelement was supplied with direct current. For different time intervals, the current at which the cooling temperature of 260 K is reached, and, accordingly, the power consumption, were calculated. The results are shown in Fig. 4. For a constant current supplying a thermoelement, there is also a time interval and a corresponding current value at which the specified temperature is reached with the lowest power consumption. Comparison of the results shown in Fig. 4 shows that when using the optimal time functions of current, cooling to a given temperature for a given time occurs with significant power savings. Depending on the time interval, power consumption savings are from 50 to 25% compared to the option of supplying the thermoelement with direct current.

One of the methods for calculating the time dependence of the current, which provides cooling to a given temperature at the maximum coefficient of performance (COP), is to use the quasi-stationary approximation [22]. This approximation is used if the volumetric heat capacity of the thermoelement material is low, which can be neglected in comparison with the heat capacity of the cooled object. If these heat capacities are comparable values, then in the calculations, the heat capacity of the material is added to the heat capacity of the object. In the quasi-stationary approximation, it is assumed that the temperature of the heat-absorbing surface of the thermoelement decreases uniformly step by step to the specified temperature. For each value of the cooling temperature, the current is determined, which provides the maximum COP value for this temperature in the steady-state mode, and the corresponding time during which the heat balance of the heat-absorbing surface of the thermoelement with the cooled object is ensured in the transient mode. The result is a time dependence of the current, for which the value of the consumed power is calculated.

An example of such a calculation of the current function and the corresponding time dependence of cooling temperature is shown in Fig. 5. The time required for cooling from 300 K to 260 K in the quasi-stationary approximation is 11 s, and the power consumption is 0.093 J. These values are significantly higher than the time of 3 s and the power consumption of 0.046 J (Fig. 4), obtained in the transient mode under the condition of optimal control of the thermoelement supply current.

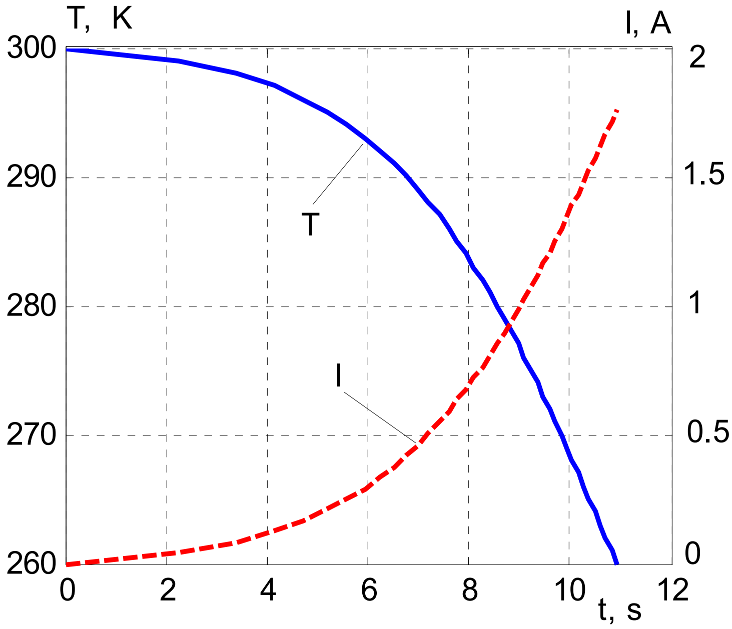


Fig. 5. The current function and the respective function of temperature reduction from 300 K to 260 K, calculated in the quasi-stationary approximation.

Thus, the comparison of the results shows that the quasi-stationary approximation is not correct enough to find the optimal time dependence of the current, which provides thermoelectric cooling with minimum power consumption.

Note that simulation of the optimal control functions of transient cooling is of great practical importance. These functions are used for the design and auto-calibration of regulators, which are necessary to ensure the operation of automatic control systems for the transient cooling process in thermoelectric devices.

Conclusions

As a result of research:

1. One of the main problems of the optimal control of transient cooling is formulated, which consists in determining the optimal time dependence of the supply current, which assures the achievement of a given cooling temperature for a given time under the condition of minimum power consumption.

2. To solve the formulated problem, a method based on the discretization of the mathematical model of transient cooling by the coordinate is proposed, which makes it possible to use the Pontryagin maximum principle for the calculation of the optimal control functions.

3. A computer simulation technique has been developed which is used to calculate the optimal functions of the supply current of thermoelements for coolers with minimum power consumption.

4. It is shown that energy saving when supplying thermoelements with an optimally time-dependent current reaches 25 – 50 % as compared to the option of direct current supply.

5. It is found that the use of the quasi-stationary approximation for calculating the optimal time dependences of the supply current of a thermoelectric cooler is incorrect.

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ОПТИМАЛЬНЕ КЕРУВАННЯ НЕСТАЦІОНАРНИМ ПРОЦЕСОМ ТЕРМОЕЛЕКТРИЧНОГО ОХОЛОДЖЕННЯ В РЕЖИМІ МІНІМАЛЬНОГО ЕНЕРГОСПОЖИВАННЯ

Сформульовано задачу оптимального керування нестаціонарним процесом термоелектричного охолодження в режимі мінімального енергоспоживання та запропоновано метод її вирішення. Розроблено алгоритм і комп'ютерний засіб, які застосовані для розрахунку оптимальних часових залежностей струму живлення термоелемента, за яких задана температура охолодження досягається за заданий час з мінімальними витратами електричної енергії. Наведено приклади комп'ютерного моделювання таких оптимальних функцій керування процесом нестаціонарного охолодження. Встановлено, що економія електроенергії за умови живлення термоелементів оптимально залежним від часу струмом досягає 25 – 50 % порівняно із варіантом живлення постійним струмом. Бібл.29 , рис. 5, табл. 1 .

Ключові слова: нестаціонарне термоелектричне охолодження, оптимальне керування, оптимальні часові залежності струму живлення термоелемента.

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ОПТИМАЛЬНОЕ УПРАВЛЕНИЕ НЕСТАЦИОНАРНЫМ ПРОЦЕССОМ ТЕРМОЭЛЕКТРИЧЕСКОГО ОХЛАЖДЕНИЯ В РЕЖИМЕ МИНИМАЛЬНОГО ЭНЕРГОПОТРЕБЛЕНИЯ

Сформулирована задача оптимального управления нестационарным процессом термоэлектрического охлаждения в режиме минимального энергопотребления и предложен метод ее решения. Разработан алгоритм и компьютерные средства, примененные для расчета оптимальных временных зависимостей тока питания термоэлемента, при которых заданная температура охлаждения достигается за заданное время с минимальными затратами электроэнергии. Приведены примеры компьютерного моделирования таких оптимальных функций управления процессом нестационарного охлаждения. Установлено, что при питании термоэлементов оптимально зависимым от времени током экономия электроэнергии достигает 25 – 50 % по сравнению с вариантом питания постоянным током. Библ.29 , рис. 5, табл. 1 .

Ключевые слова: нестационарное термоэлектрическое охлаждение, оптимальное управление, оптимальные временные зависимости тока питания термоэлемента.

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