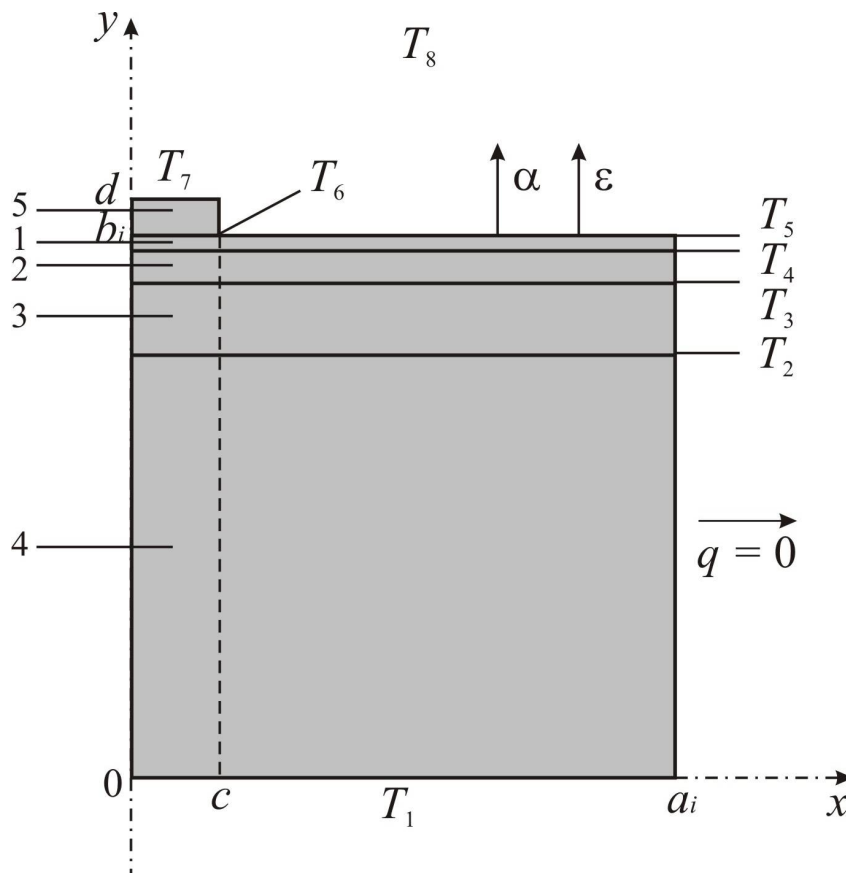


Physical model

According to physical 2D model with axial symmetry (Fig. 1), the biological tissue of human body is a structure of three skin layers (epidermis 1, dermis 2, subcutis 3) and the internal biological tissue 4, and is characterized by the following thermophysical properties: thermal conductivity κ_i , specific heat C_i , density ρ_i , blood perfusion rate ω_{bi} , blood density ρ_b , blood temperature T_b , specific heat of blood C_b and specific heat release Q_{meti} due to metabolic processes and latent phase transition heat L . The thermophysical properties of the skin and the biological tissue of human body in the normal and frozen states are given in [20 – 27]. The respective layers of the biological tissue 1-4 are considered as volumetric heat sources q_i , where:

$$q_i = Q_{meti} + \rho_b \cdot C_b \cdot \omega_{bi} \cdot (T_b - T), \quad i = 1..4. \quad (1)$$

On the surface of the skin there is a cooling element 5. The geometric dimensions of each such layer 1-4 are a_i , b_i , and, respectively, of cooling element 5 – c , d . The temperatures at the boundaries of the respective layers 1-4 and cooling element 5 are T_1 , T_2 , T_3 , T_4 , T_5 , T_6 . The temperature inside the biological tissue is $T_1 = +37^\circ\text{C}$. The temperature of the cooling element is $T_7 = -50^\circ\text{C}$. The ambient temperature is $T_8 = +22^\circ\text{C}$. The surface of human skin with a temperature of T_6 is in a state of heat exchange with the environment (heat transfer coefficient α and emissivity ε) at temperature T_8 . The lateral surface of the skin is adiabatically isolated.



*Fig.1. Physical 2D model of human skin with axial symmetry:
 1 – epidermis, 2 – dermis, 3 – subcutis; 4 – internal biological tissue,
 5 – cooling element*

Mathematical model

In general, the equation of heat transfer in the biological tissue is given by [20-27]:

$$C_i \cdot \frac{\partial T}{\partial t} = \nabla \cdot (\kappa_i \cdot \nabla T) + \rho_b \cdot C_b \cdot \omega_{bi} \cdot (T_b - T) + Q_{meti}, \quad i = 1..4, \quad (2)$$

where C , κ are specific heat and thermal conductivity of the biological tissue, ρ_b is blood density, C_b is specific heat of blood, ω_b is blood perfusion of corresponding layers, T_b is blood temperature, T is temperature of the biological tissue; Q_{met} is heat which is released due to metabolic processes in each layer.

The term on the left side of equation (2) is the rate of change of thermal energy contained in a unit volume of the biological tissue. The three terms on the right side of this equation represent, respectively, the rate of change of thermal energy due to thermal conductivity, blood perfusion, and metabolic heat.

The equation of heat transfer in the biological tissue (2) is solved with the corresponding boundary conditions. The temperature on the surface of cooling element is $T_7 = -50^\circ\text{C}$. The temperature inside the biological tissue is $T_1 = +37^\circ\text{C}$. The lateral surfaces of the biological tissue are adiabatically isolated ($q = 0$), and the upper surface of the skin is in a state of heat exchange (heat transfer coefficient α and emissivity ε) with the environment at a temperature of T_8 .

$$q(x, y, t) \Big|_{\substack{c \leq x \leq a \\ y = b_1}} = \alpha \cdot (T_8 - T_5) + \varepsilon \cdot \sigma \cdot (T_8^4 - T_5^4), \quad (3)$$

where α is coefficient of convective heat exchange of the surface of the skin with the environment, ε is emissivity, σ is the Boltzmann constant, T_5 is the temperature of the biological tissue surface, T_8 is ambient temperature ($T_8 = +22^\circ\text{C}$).

At the initial time $t = 0$ s, it is believed that the temperature in the bulk of the biological tissue is $T = +37^\circ\text{C}$, that is, the initial conditions for solving equation (2) are as follows:

$$T(x, y, 0) = T_b. \quad (4)$$

As a result of solving the initial boundary value problem (2) - (4), the distributions of temperature $T(x, y, t)$ and heat fluxes in the respective skin layers are determined at an arbitrary time. As an example, in this paper we consider a case in which the temperature of cooling element is $T_7 = -50^\circ\text{C}$. However, it should be noted that the proposed method allows considering cases where the temperature of cooling element $T_f(t)$ changes in any temperature range or according to a predetermined function.

During the freezing, the cells will undergo a phase change at the freezing point, with the loss of the phase transition heat (L), and the temperature in these cells will not change. The phase transition in the biological cells occurs in the temperature range $(-1 \div -8)^\circ\text{C}$. The properties of the skin and the biological tissue in the normal and frozen states are shown [20 – 27]. In the temperature range $(-1 \div -8)^\circ\text{C}$, when cells are frozen, the heat of the phase transition is absorbed, which can be simulated by adding an appropriate value to the heat capacity [26, 27].

When the biological tissue is frozen, the vessels in the capillaries are narrowed to freeze all blood in the capillaries, and the value ω_{bi} tends to zero. In addition, the cells will not be able to generate metabolic heat when frozen and Q_{met} will be zero at a temperature below zero.

In the frozen state the properties of the skin and the biological tissue will have the following values (5) – (8):

$$C_i = \begin{cases} C_1 & T \geq -1^\circ C \\ \frac{L}{-1 - (-8)} + \frac{C_1 + C_2}{2} & -8^\circ C \leq T \leq -1^\circ C \\ C_2 & T \leq -8^\circ C \end{cases} \quad (5)$$

$$\kappa_i = \begin{cases} \kappa_1 & T \geq -1^\circ C \\ \frac{\kappa_1 + \kappa_2}{2} & -8^\circ C \leq T \leq -1^\circ C \\ \kappa_2 & T \leq -8^\circ C \end{cases} \quad (6)$$

$$Q_{met,i} = \begin{cases} 420 & T \geq -1^\circ C \\ 0 & -8^\circ C \leq T \leq -1^\circ C \\ 0 & T \leq -8^\circ C \end{cases} \quad (7)$$

$$\omega_{b_i} = \begin{cases} 0,0005 & T \geq -1^\circ C \\ 0 & -8^\circ C \leq T \leq -1^\circ C \\ 0 & T \leq -8^\circ C \end{cases} \quad (8)$$

Computer model

A computer model of human skin was created on the surface of which there is a cooling element. To build a computer model, the Comsol Multiphysics application package was used [28], which makes it possible to simulate thermophysical processes in the biological tissue taking into account blood circulation, heat transfer, metabolic processes, and phase transition.

The distribution of temperatures and heat fluxes in the human skin and, accordingly, the biological tissue was calculated by the finite element method, the essence of which is that the object under study is divided into a large number of finite elements, and in each of them the value of a function is sought that satisfies given second-order differential equations with the corresponding boundary conditions. The accuracy of solving the problem depends on the level of partitioning and is ensured by the use of a large number of finite elements [28].

As an example, Figs. 2-3 show the distribution of temperature and isothermal surfaces in the bulk of human skin, on the surface of which a cooling element is placed at a temperature of $T = -50^\circ C$.

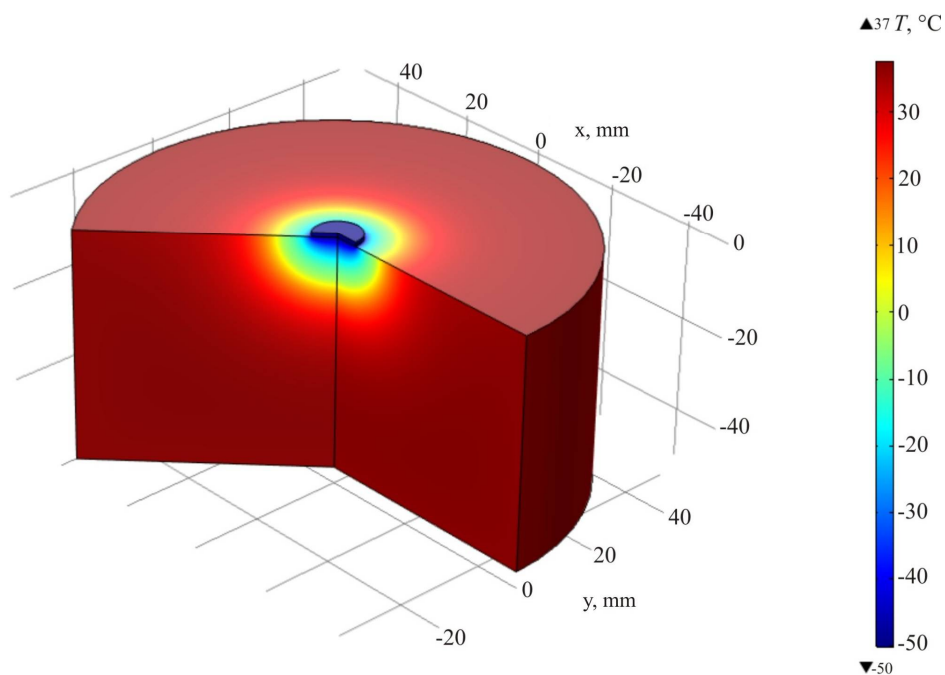


Fig.2. Temperature distribution in the bulk of human skin on the surface of which there is a cooling element at a temperature of $T = -50^{\circ}\text{C}$

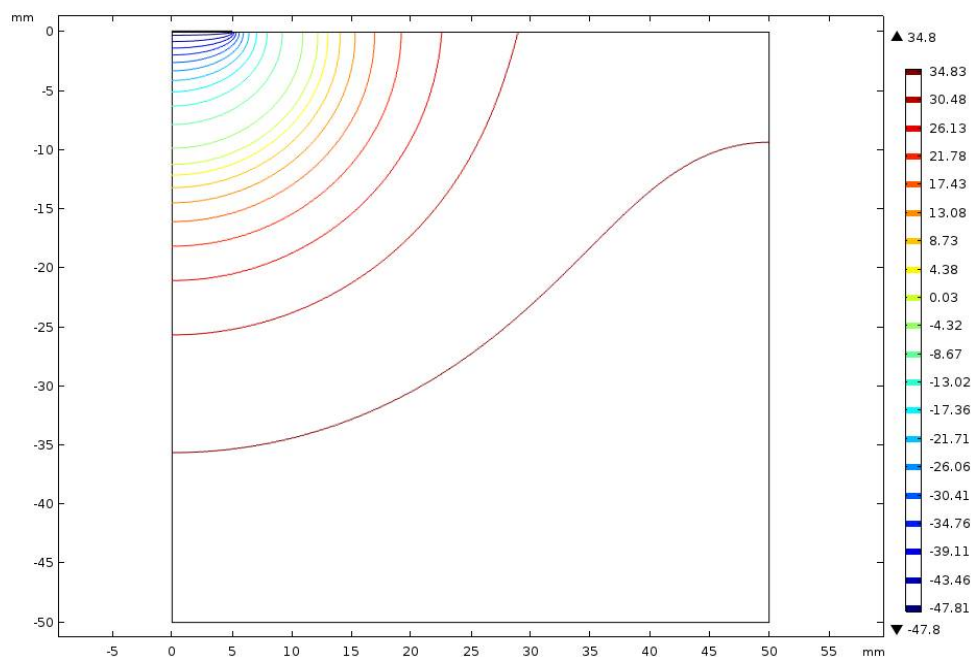
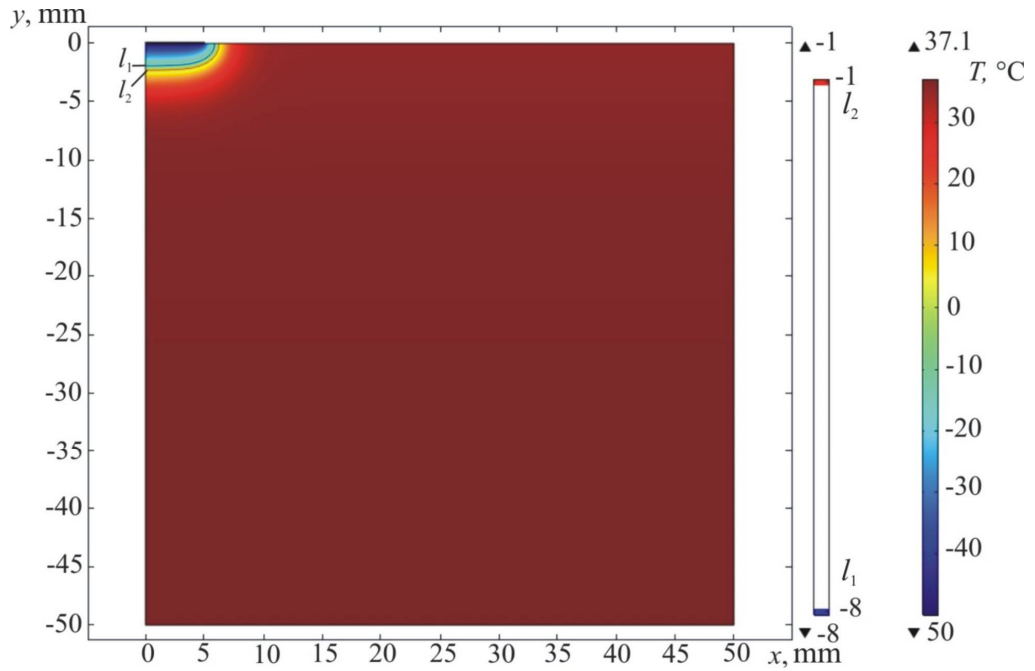


Fig. 3. Isothermal surfaces in the bulk of human skin on the surface of which there is a cooling element at a temperature of $T = -50^{\circ}\text{C}$

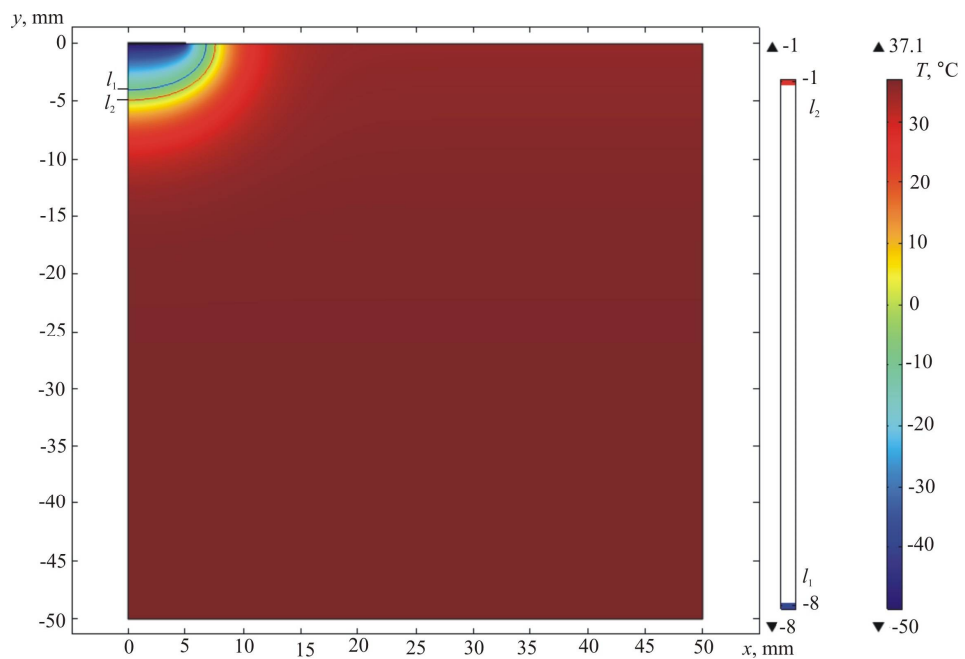
Computer simulation results

Figs.4 a, b, c, d, e, f show temperature distributions in the section of human skin on the surface of which there is a cooling element at a temperature of $T = -50^{\circ}\text{C}$ at different time moments

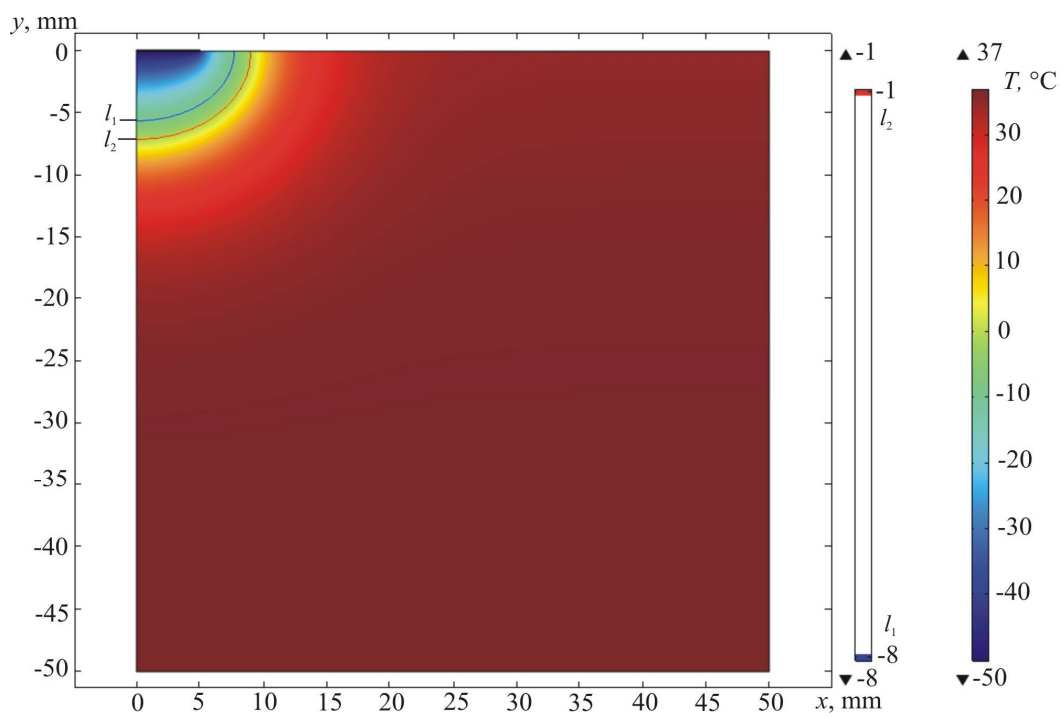
$t = 10, 60, 180, 300, 600, 1200$ s. In so doing, l_1 is temperature level $T = -8^\circ\text{C}$ and l_2 is temperature level $T = -1^\circ\text{C}$.



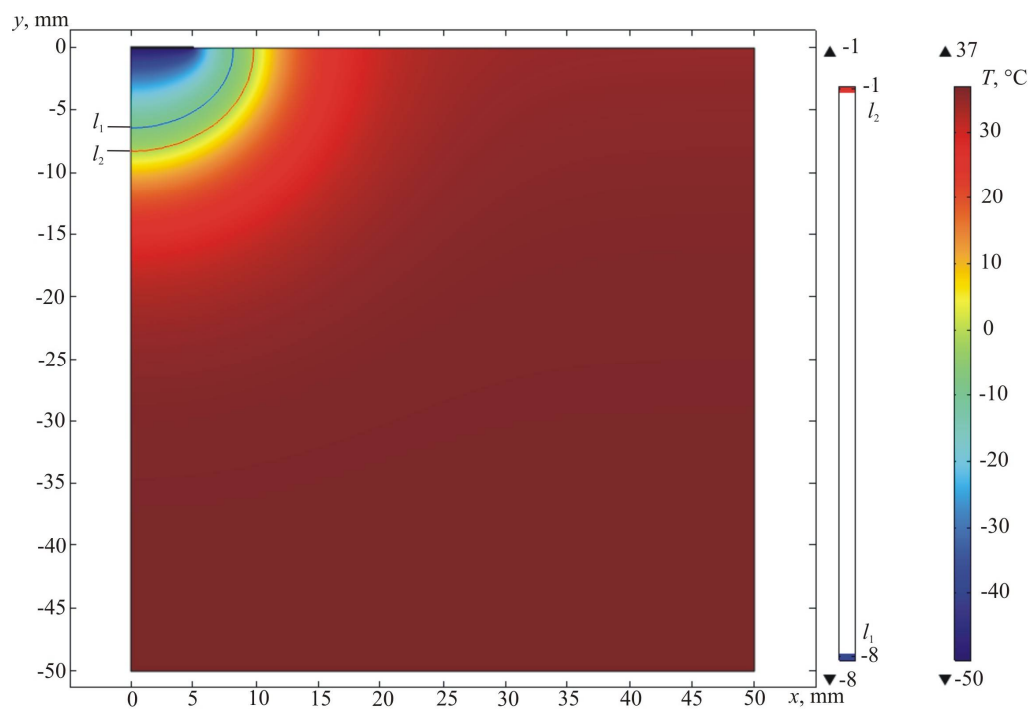
a) $t = 10$ s



b) $t = 60$ s



c) $t = 180$ s



d) $t = 300$ s

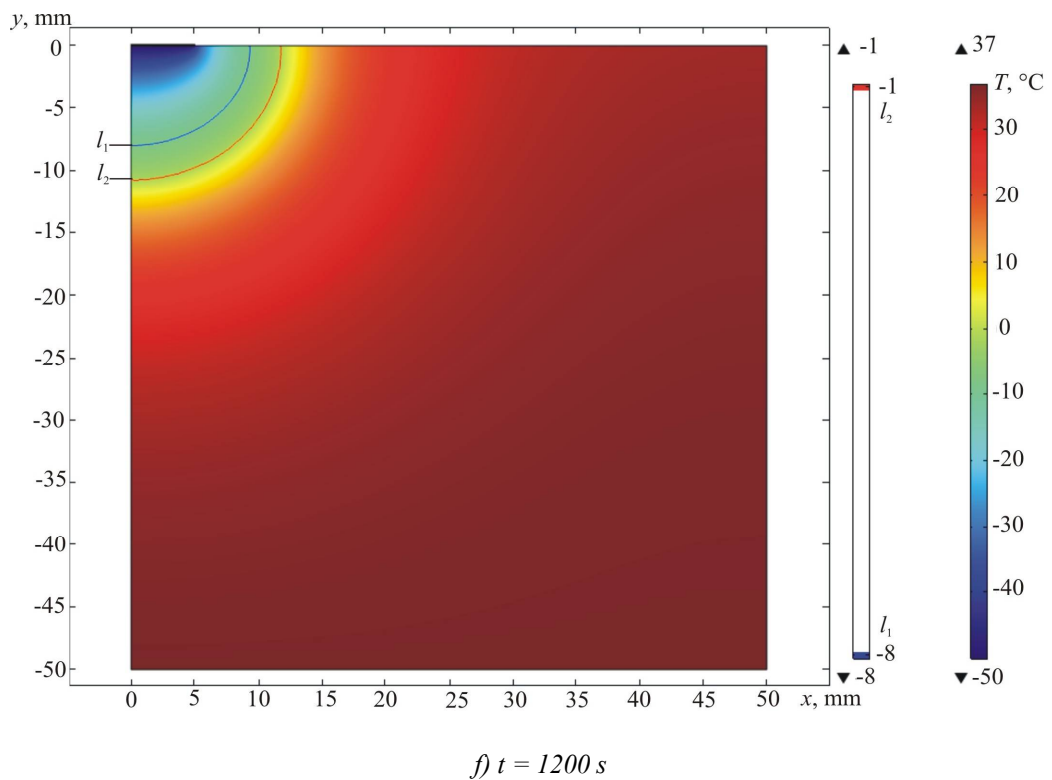
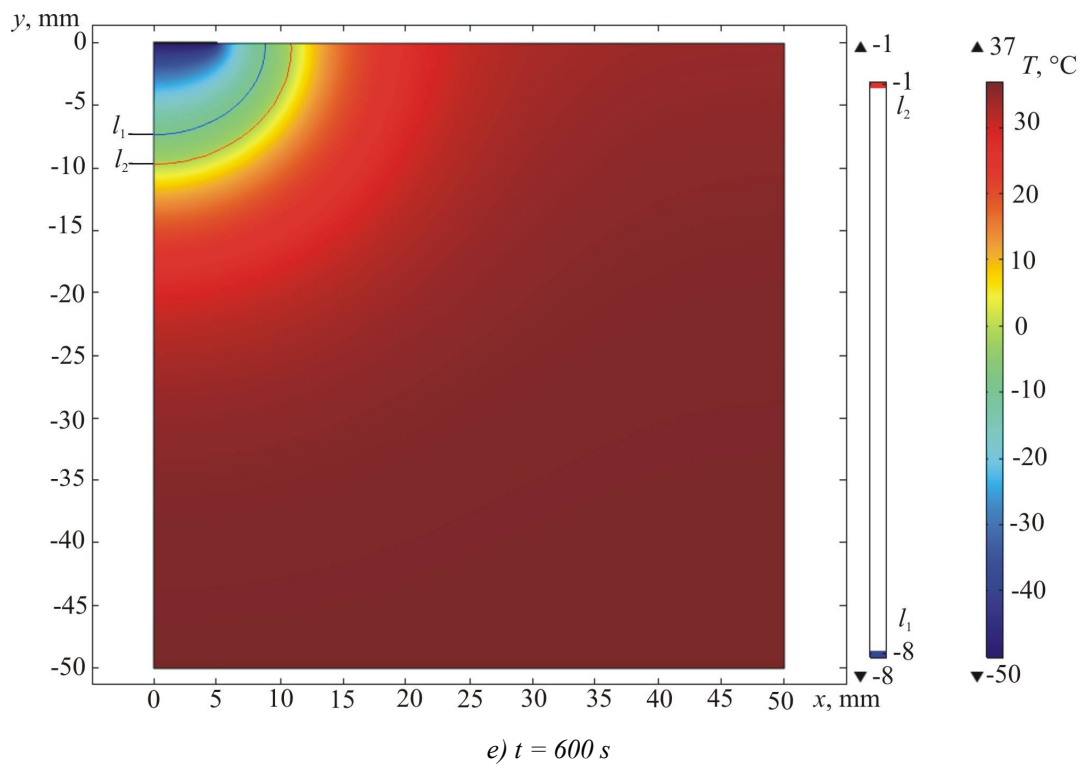


Fig.4 a, b, c, d, e, f. Temperature distributions in the section of human skin on the surface of which there is a cooling element at a temperature of $T = -50^\circ\text{C}$, at different time moments:

a) $t = 10$ s, b) $t = 60$ s, c) $t = 180$ s, d) $t = 300$ s, e) $t = 600$ s, f) $t = 1200$ s,
where l_1 is temperature level $T = -8^\circ\text{C}$ and l_2 is temperature level $T = -1^\circ\text{C}$

Fig. 5 shows the dependence of the movement of the phase transition zone (crystallization zone of the biological tissue) on the time of temperature exposure. From Fig. 5 it is obvious that the maximum freezing depth of human skin and, accordingly, the biological tissue is about $l \approx 10$ mm at a temperature of cooling element $T = -50$ °C.

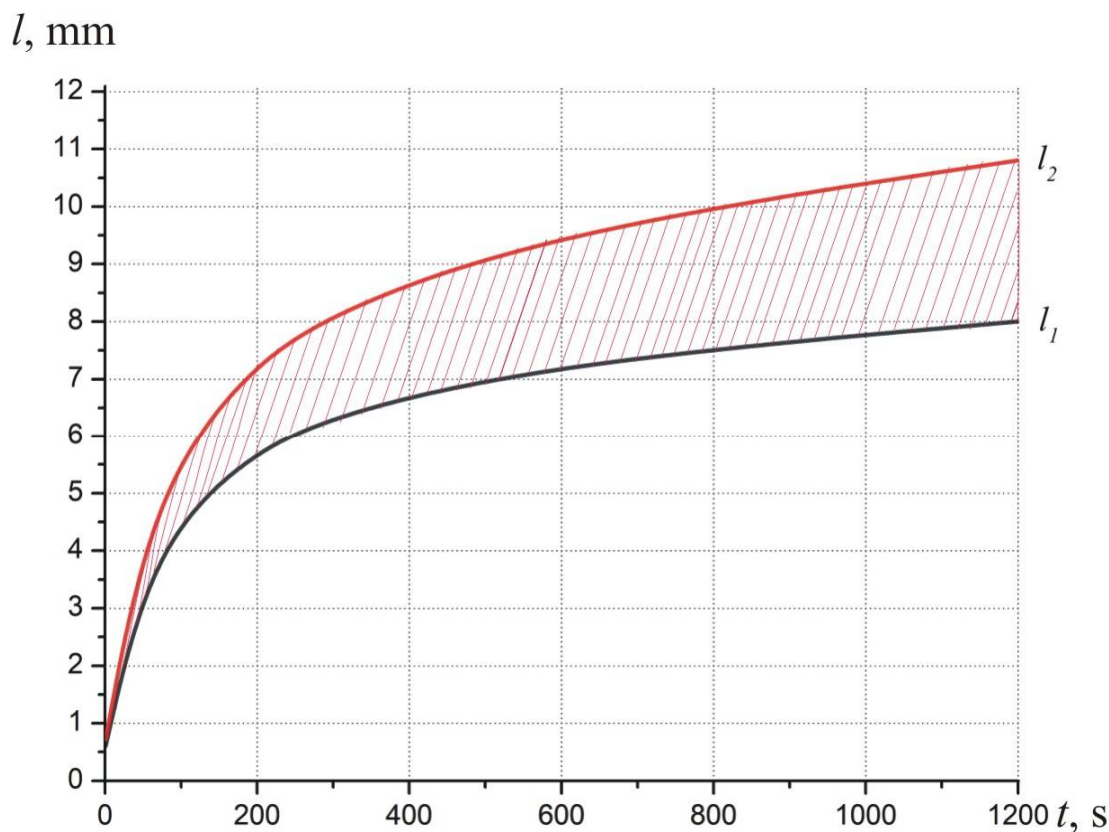
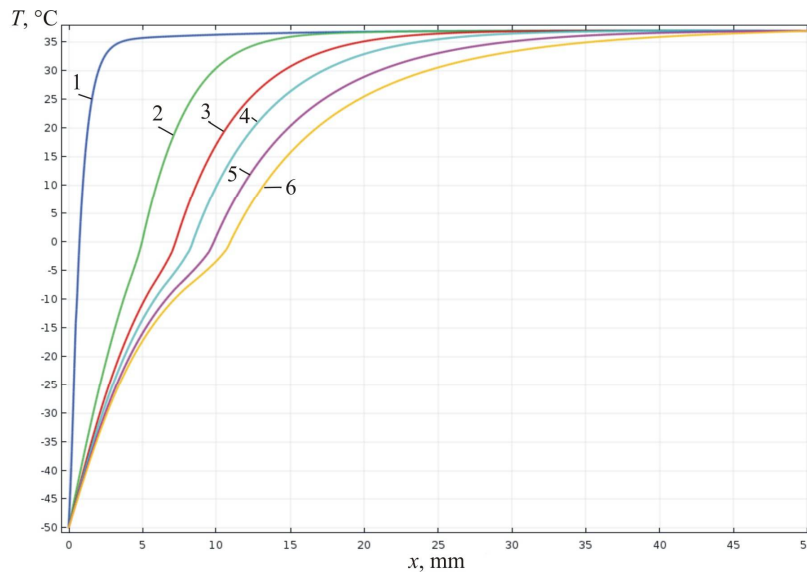


Fig. 5. Dependence of the movement of the phase transition zone (crystallization zone of the biological tissue) on the time of temperature exposure at a temperature of cooling element $T = -50$ °C: l_1 – temperature level $T = -8$ °C and l_2 – temperature level $T = -1$ °C

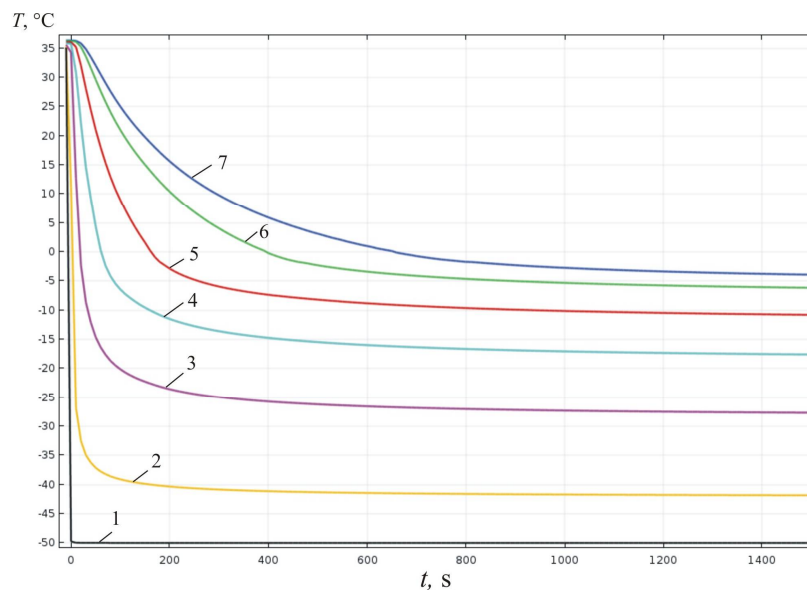
Using computer simulation, we determined the dependence of the depth of freezing of human skin on temperature at different times (Fig. 6) and on the time of temperature exposure at a temperature of cooling element $T = -50$ °C (Fig. 7).

Figs. 6, 7 show that at $t = 60$ s the biological tissue is cooled to a temperature of $T = 10$ °C at a depth of $l \approx 3.5$ mm, at $t = 180$ s - at a depth of $l \approx 5$ mm, and at $t = 600$ s - at a depth of $l \approx 7$ mm and at $t = 1200$ s - at a depth of $l \approx 7.5$ mm.

It is established that with increasing temperature exposure, a deeper cooling of human skin is achieved. That is, with a prolonged temperature exposure ($T = -50$ °C), a destruction of the corresponding area of human skin can be achieved.



*Fig. 6. Temperature distribution in human skin at different time moments of temperature exposure:
 1 – $t = 1$ s; 2 – $t = 60$ s; 3 – $t = 180$ s; 4 – $t = 300$ s; 5 – $t = 600$ s; 6 – $t = 1200$ s*



*Fig. 7. Temporal dependence of temperature at different depth h of human skin
 at a temperature of cooling element $T = -50^{\circ}\text{C}$: 1 – $h = 0$; 2 – $h = 1$ mm; 3 – $h = 3$ mm;
 4 – $h = 5$ mm; 5 – $h = 7$ mm; 6 – $h = 9$ mm; 7 – $h = 10$ mm*

Thus, a technique was developed for taking into account the phase transition in human skin during computer –aided simulation of cryodestruction process, which makes it possible to predict the results of local temperature effect on the biological tissue and to determine the temperature and heat flux distributions at any time moment with a predetermined arbitrary time function of change in the temperature of cooling element $T_f(t)$ [29].

It should be noted that the obtained results make it possible to predict the depth of freezing of the skin, and, accordingly, the biological tissue at a given temperature exposure, taking into account the phase transition to achieve the maximum effect during cryodestruction of human skin. They are also necessary for the design of thermoelectric refrigerators for cryodestruction of the skin and providing the necessary cooling modes.

Conclusion

1. A physical, mathematical and computer models of human skin, on the surface of which there is a cooling element at a temperature of $T = -50^{\circ}\text{C}$ were created with regard to thermophysical processes, blood circulation, heat transfer, metabolic and phase transition processes.
2. Using computer simulation, the distribution of temperature and heat fluxes in various skin layers was determined taking into account the phase transition in the process of cryodestruction of human skin. The dependence of the freezing depth of human skin on the temperature of cooling element and the time of the temperature exposure was established. The maximum freezing depth of the skin was determined which is $l \approx 10$ mm at a temperature of cooling element $T = 50^{\circ}\text{C}$.

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МЕТОДИКА ВРАХУВАННЯ ФАЗОВОГО ПЕРЕХОДУ В БІОЛОГІЧНІЙ ТКАНИНІ ПРИ КОМП'ЮТЕРНОМУ МОДЕЛЮВАННІ ПРОЦЕСУ КРІОДЕСТРУКЦІЇ

У роботі наведено результати комп'ютерного моделювання процесу кріодеструкції шкіри людини з врахуванням теплофізичних процесів, кровообігу, теплообміну, процесів метаболізму та фазового переходу. Побудовано фізичну, математичну та комп'ютерну моделі шкіри людини, на поверхні якої знаходиться охолоджуючий елемент при температурі -50°C. Визначено розподіли температури і теплових потоків у шкірі людини в режимі охолодження. Отримані результати дають можливість прогнозувати глибину промерзання шкіри і, відповідно, біологічної тканини при заданому температурному впливі. Бібл. 29, рис. 7.

Ключові слова: шкіра людини, температурний вплив, кріодеструкція, фазовий перехід, комп'ютерне моделювання.

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КОМП'ЮТЕРНОЕ МОДЕЛИРОВАНИЕ ПРОЦЕССА КРИОДЕСТРУКЦИИ КОЖИ ЧЕЛОВЕКА ПРИ ТЕРМОЭЛЕКТРИЧЕСКОМ ОХЛАЖДЕНИИ

В работе приведены результаты компьютерного моделирования процесса криодеструкции кожи человека с учетом теплофизических процессов, кровообращения, теплообмена, процессов метаболизма и фазового перехода. Построены физическая, математическая и компьютерная модели кожи человека, на поверхности которой находится охлаждающий

элемент при температуре $-50\text{ }^{\circ}\text{C}$. Определено распределения температуры и тепловых потоков в коже человека в режиме охлаждения. Полученные результаты дают возможность прогнозировать глубину промерзания кожи и, соответственно, биологической ткани при заданном температурном воздействии. Библ. 28, рис. 7.

Ключевые слова: кожа человека, температурное влияние, криодеструкция, фазовый переход, компьютерное моделирование.

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Мыцканюк Н.В.

ON THE TEMPERATURE DEPENDENCES OF THERMOELECTRIC CHARACTERISTICS OF THERMOELECTRIC MATERIAL-METAL TRANSIENT LAYER WITHOUT REGARD TO PERCOLATION EFFECT

The basic relationships are obtained by calculation, which determine the temperature dependences of thermoelectric characteristics of thermoelectric material-metal transient contact layers without regard to percolation theory. Specific quantitative results and plots of the temperature dependences of the electrical and thermal contact resistances, the thermoEMF, the power factor, and the dimensionless thermoelectric figure of merit are given for bismuth telluride – nickel contact pair. It has been established that with uneven distribution of metal atoms in the temperature range of 200 - 400 K, the intensity of metal atoms entering transient layer, which corresponds to a change in the distribution of metal atoms by the thickness of transient layer from linear to square and the thickness range of transient layer from 20 to 150 μm , the electrical contact resistance varies from $1.8 \cdot 10^{-7}$ to $4.8 \cdot 10^{-6} \text{ Ohm}\cdot\text{cm}^2$, the thermal contact resistance - from 0.022 to 0.35 $\text{K}\cdot\text{cm}^2/\text{W}$, the thermoEMF - from 155 to 235 $\mu\text{V}/\text{K}$, the power factor - from $1.6 \cdot 10^{-4}$ to $2.9 \cdot 10^{-4} \text{ W}/(\text{m}\cdot\text{K}^2)$, the dimensionless thermoelectric figure of merit - from 0.55 to 1.7. Bibl. 34, Fig. 21.

Key words: thermoelectric material-metal contact, near-contact transient layer, electrical contact resistance, thermal contact resistance, thermoEMF, power factor, dimensionless thermoelectric figure of merit, temperature dependences.

Introduction

The efficiency of thermoelectric modules, which is mainly determined by the figure of merit of thermoelement leg materials, essentially depends on the electrical contact resistance at the boundaries between semiconductor materials of legs with metal interconnect layers. Contact resistance is one of the main reasons for the fact that in thermoelectric coolers and generators the properties of materials are not fully realized [1 – 3]. It is well known that the Joule heat, which is released on contact resistances, reduces the energy efficiency of thermoelectric converters and leads to the dependence of their characteristics on the height of thermoelement legs [4]. The influence of contact resistance on the characteristics of thermoelectric devices becomes more significant under conditions of miniaturization of thermoelectric legs, when the thickness of thermoelectric material-metal transient contact layers along the electric current directions becomes comparable with the height of thermoelectric legs [5, 6] Miniaturization of thermoelectric power converters is a modern trend of their improvement [7 – 12], aimed primarily at reducing the expenses for thermoelectric materials and thereby cheapening the thermoelectric modules. Therefore, studies of the electrical properties of TEM-metal transient layers, aimed at reducing the contact