DOI: 10.63527/1607-8829-2026-2-15-39

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Theory of Dipole-Exchange Spin Waves in a Ferromagnetic Nanotube in the Presence of a Thermoelectrically Induced Spin-Transfer Torque

A theoretical study of dipole–exchange spin waves in a conducting ferromagnetic nanotube subjected to a longitudinal temperature gradient is presented. The temperature gradient via the Seebeck effect generates an electric current, which becomes spin-polarized in the ferromagnet and gives rise to a spin-transfer torque acting on the magnetization. The magnetization dynamics is described within the Landau–Lifshitz–Gilbert framework augmented by the Zhang–Li torque terms corresponding to adiabatic and nonadiabatic spin transport. An analytical dispersion relation for spin waves in a thin-walled nanotube with uniaxial anisotropy is derived, taking into account both exchange and dipolar interactions. It is shown that the thermoelectrically induced spin-polarized current leads to a Doppler-like shift of the spin-wave spectrum and modifies the effective damping. A critical temperature gradient is obtained at which the nonadiabatic torque compensates the intrinsic Gilbert damping, resulting in the onset of spin wave generation. Numerical estimates for Permalloy nanotubes demonstrate that the effect can be significant for experimentally accessible parameters. The results reveal a direct coupling between thermoelectric charge transport and spin-wave dynamics in curved magnetic nanostructures, highlighting the potential of ferromagnetic nanotubes as elements of spin-caloritronic and thermoelectric devices.

Keywords: spin wave; nanomagnetism; dipole-exchange theory; ferromagnetic nanotube; uniaxial magnetic anisotropy; spin-polarized current; thermoelectricity; spin transfer; Seebeck effect.

Citation: V.V. Kulish, S.I. Yeromin (2026). Theory of Dipole-Exchange Spin Waves in a Ferromagnetic Nanotube in the Presence of a Thermoelectrically Induced Spin-Transfer Torque. *Journal of Thermoelectricity*, (2), 15–39. https://doi.org/10.63527/1607-8829-2026-2-15-39

Received: 20.04.2026; Revised: 05.05.2026; Published: 30.06.2026

Introduction

Spin waves (magnons) in magnetically ordered media remain a subject of sustained theoretical and experimental interest due to their fundamental importance and their potential applications (in particular, in magnonics and spintronics) [1, 2]. In nanoscale magnetic systems, the properties of spin waves are strongly affected by geometry, boundary conditions, exchange interaction, dipole–dipole interaction, magnetic anisotropy, and dissipation effects [1–3]. In particular, nanosystems possessing one-axis translational symmetry – such as magnetic nanowires and nanotubes – represent an important class of waveguiding structures for spin excitations.

Magnetic nanotubes have been synthesized relatively recently and have already demonstrated promising applications in nanomagnetism and magnetobiology [4, 5]. However, compared to thin films and planar multilayers, spin-wave phenomena in nanotubes remain less extensively studied. In previous works by the authors, dipole-exchange spin waves in ferromagnetic nanotubes of circular and elliptic cross-sections in the presence of a spin current [6, 7] were investigated within the framework of the linearized Landau-Lifshitz equation in the magnetostatic approximation. In these studies, the influence of a spin-polarized electric current was incorporated via the Slonczewski-Berger spin-transfer torque term [8, 9], as the spin current was assumed to be generated by an externally applied electric current. However, in recent years considerable attention has been devoted to thermally driven spin transport phenomena, including Seebeck-induced spin-polarized charge current, spin Seebeck effect and related spin-caloritronic effects [10–12]. In a conducting ferromagnet subjected to a temperature gradient, an electric current arises due to the Seebeck effect. If the conduction electrons are spin-polarized, this thermoelectric current is accompanied by a spin current, which can exert a spin-transfer torque on the local magnetization [13, 14]. Thus, a temperature gradient can indirectly influence spin-wave dynamics through a Seebeck-induced spin current.

From the viewpoint of thermoelectricity, magnetic nanostructures often operate under conditions of non-uniform temperature distributions [15]. However, to the best of our knowledge, a systematic theory of dipole-exchange spin waves in a ferromagnetic nanotube driven by a thermoelectrically induced spin-polarized current is still lacking.

From an applications viewpoint, the possibility of controlling the spin-wave spectrum by a temperature gradient is relevant to several current directions in magnonics [1, 16]. Frequency tuning and nonreciprocal propagation are important for reconfigurable magnonic waveguides, directional couplers, and diode-like elements, while control of the effective damping is directly related to spin-wave amplification, loss compensation, and auto-oscillatory magnon sources. These functionalities are, in turn, of interest for spin-wave logic, interferometric processing, reservoir computing, and other low-power magnonic information-processing schemes [1, 16–19]. Recent roadmaps on magnonics and 3D nanomagnetism also emphasize cylindrical and curvilinear magnetic architectures as promising platforms for waveguiding, symmetry breaking, and nonreciprocal spin-wave transport.

The aim of the present work is to develop a theory of dipole-exchange spin waves in a uniaxial ferromagnetic nanotube in the presence of a spin current induced by an axial temperature gradient. We consider a one-layer ferromagnetic nanotube in which the temperature gradient generates an electric current via the Seebeck effect; this current, being spin-polarized, produces a spin-transfer torque that is incorporated into the linearized Landau-Lifshitz equation. Exchange interaction, dipole-dipole magnetic interaction, uniaxial anisotropy, and Gilbert damping are taken into account. Using the magnetostatic approximation, we derive the equation for the magnetic potential of spin excitations and obtain the dispersion relation for such waves.

Special attention is paid to the influence of the temperature gradient on the real and imaginary parts of the spin-wave frequency. We show that the temperature gradient modifies both the real part of the frequency (Doppler shift) and the effective damping of the spin wave; we derive the condition under which the thermoelectrically induced spin-transfer torque compensates intrinsic dissipation, leading to spin-wave amplification or generation. In this way, the present work establishes a theoretical connection between thermoelectric transport and spin-wave dynamics in cylindrical ferromagnetic nanostructures.

Problem statement

Let us consider a ferromagnetic nanotube whose length is much larger than its outer radius, so that the system can be regarded as possessing a one-dimensional translational symmetry along its axis. The nanotube is assumed to be composed of a uniaxial ferromagnet of the "easy-axis" type. The easy axis of the ferromagnet is directed along the symmetry axis of the nanotube. For convenience, we choose this direction as the Oz axis of the cylindrical coordinate system, see Fig. 1.

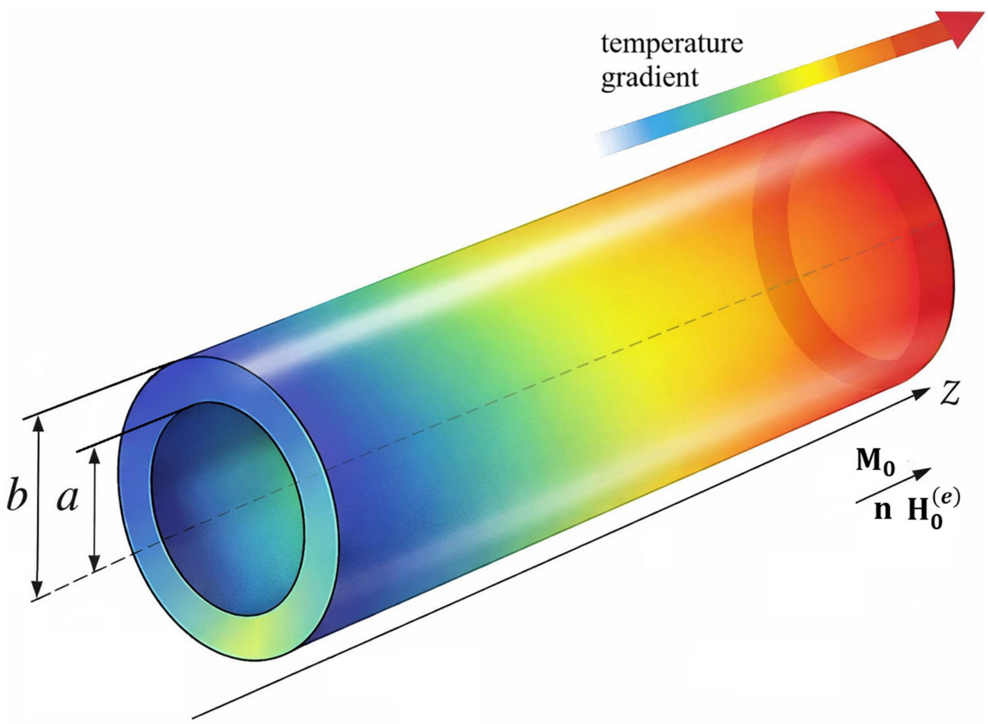


Fig. 1. Nanotube investigated in the paper

The nanotube is placed in an external homogeneous magnetic field $\mathbf{H}_0^{(e)}$ directed along the same axis. Under these conditions, the equilibrium magnetization $\mathbf{M}_0$ of the ferromagnet is also directed along the nanotube axis. We assume that the ferromagnetic material is characterized by the exchange interaction constant α, the uniaxial anisotropy parameter β, the saturation magnetization M_0, and the gyromagnetic ratio γ. Dissipative effects are taken into account using the Gilbert damping parameter α_G.

We consider a temperature gradient ∇T applied along the nanotube axis. The nanotube is considered to be a part of a closed thermoelectric circuit, so that the temperature gradient generates a stationary electric current via the Seebeck effect [13, 14]. Since the electrical transport in a ferromagnet is spin dependent, the thermoelectric transport must be described within the framework of the two-current model, in which electrons with spin parallel and antiparallel to the magnetization propagate in separate transport channels characterized by different conductivities and thermopowers.

In this two-current model for a closed circuit, the electric current densities for the two spin channels are

$$j_\uparrow = -\sigma_\uparrow S_\uparrow \nabla T, \qquad j_\downarrow = -\sigma_\downarrow S_\downarrow \nabla T,$$

where $\sigma_\uparrow$, $\sigma_\downarrow$ and $S_\uparrow$, $S_\downarrow$ are the conductivities and Seebeck coefficients for the majority and minority spin channels, respectively. Here, we use the ideal short-circuit approximation and neglect the self-consistent electrostatic field. The total electric current density in the nanotube is $J = j_\uparrow + j_\downarrow$, while the corresponding spin current (to be exact, spin-polarized charge current) density $J_s = j_\uparrow - j_\downarrow$. As a result, the thermoelectric current in a conducting ferromagnet is generally spin-polarized. The degree of spin polarization of the thermoelectric current can be characterized by the parameter

$$P_T = \frac{j_\uparrow - j_\downarrow}{j_\uparrow + j_\downarrow}.$$

Thus, the thermoelectric current flowing along the nanotube axis carries spin angular momentum and may interact with the magnetization of the ferromagnet through the spin-transfer torque mechanism.

A possible additional contribution to the spin transport in the considered system is the purely magnon spin current generated by the temperature gradient (spin Seebeck effect). In contrast to the electron-mediated spin current discussed above, this current is not associated with charge transport and is carried by nonequilibrium magnons. In the diffusive approximation, such a current is usually described by the relation

$$j_m = -\sigma_m \nabla \mu_m - \zeta_m \nabla T,$$

where σ_m is the magnon spin conductivity, μ_m is the magnon chemical potential, and ζ_m is the magnon spin Seebeck coefficient. If magnon accumulation is weak, this expression reduces to

$$j_m \approx -\zeta_m \nabla T.$$

However, unlike the spin-polarized electric current, the magnon current contributes to the local magnetization dynamics only through its nonuniformity, i.e. through $\mathrm{div} j_m$, or via interfacial spin conversion. Therefore, in a sufficiently homogeneous conducting nanotube with an approximately uniform axial temperature gradient, its influence is additionally reduced by the factor d/L_{rel}, where d is the nanotube wall thickness and L_{rel} is the characteristic magnon relaxation length. The magnon contribution may thus be neglected when

$$\frac{2e}{\hbar}\frac{\zeta_m}{P_T \sigma S}\frac{d}{L_{\mathrm{rel}}} \ll 1,$$

where P_T, σ, and S characterize the thermoelectrically induced spin-polarized electric current. *This condition is typically fulfilled for thin metallic nanotubes with high electrical conductivity,* moderate thermoelectric spin polarization, and weak axial nonuniformity of the magnon subsystem. Hence, in the first approximation, it is reasonable to restrict the analysis to the electron-mediated thermoelectric spin-transfer mechanism.

Let us note that the presence of the above-mentioned longitudinal electric current generates a magnetic self-field that should, in general case, be taken into account by adding corresponding term to the external magnetic field $\mathbf{H}_0^{(e)}$. This self-field is, evidently, azimuthal and non-uniform (growing monotonically from 0 on the internal nanotube surface to the maximum value $H_\varphi^{max} = \frac{2\pi J_z (b^2 - a^2)}{bc}$ on the external surface, here *c* is the light velocity in vacuum). Let us assume that the external magnetic field and equilibrium magnetization $\mathbf{M_0}$ both have big enough values so that $H_\varphi^{max} \ll H_0^{(e)}$, $H_\varphi^{max} \ll 4\pi M_0$. Under these conditions the self-field can be neglected, which simplifies the research essentially. After taking into account the relation $J = -(\sigma_\uparrow S_\uparrow + \sigma_\downarrow S_\downarrow)\nabla T = -\sigma S \nabla T$ (here σ is the total electric conductivity and S is the total Seebeck coefficient), one can obtain that the self-field can be neglected if the relations for the temperature gradient

$$\frac{2\pi\sigma S(b^2 - a^2)}{bc}\nabla T \ll H_0^{(e)}, \qquad \frac{2\pi\sigma S(b^2 - a^2)}{bc}\nabla T \ll 4\pi M_0$$

fulfill.

Let us study spin waves propagating along the nanotube axis of the above-described nanotube. Since the system under consideration has nanoscale dimensions, the exchange interaction can significantly influence the spin-wave spectrum. Therefore, the description of spin waves must include both the magnetic dipole-dipole interaction and the exchange interaction. The corresponding excitations are thus described within the framework of dipole-exchange spin-wave theory.

Let us use linearized spin wave theory and, thus, consider small deviations of the magnetization from its equilibrium value. The magnetization of the spin wave **m** and the magnetic field of the wave **h** are assumed to be small perturbations of the equilibrium magnetization $\mathbf{M_0}$ and the equilibrium internal magnetic field $\mathbf{H}_0^{(i)}$, respectively. Thus, the total magnetization and internal magnetic field can be written as

$$\mathbf{M} = \mathbf{M}_0 + \mathbf{m}, |\ \mathbf{m}\ | \ll |\ \mathbf{M}_0\ |$$

$$\mathbf{H}^{(i)} = \mathbf{H}_0^{(i)} + \mathbf{h}, |\ \mathbf{h}\ | \ll |\ \mathbf{H}_0^{(i)}\ |$$

which allows applying the linearized theory of spin waves.

In the theoretical description of the system, the influence of the thermoelectric current is taken into account through the spin-transfer torque term in the Landau-Lifshitz equation. In contrast to the case of externally injected electric current considered in earlier studies [6, 7], the current in the present system arises due to the temperature gradient via the Seebeck effect. Consequently, the density of the electric current in the nanotube is determined by the thermoelectric transport parameters of the material.

The main goal of the proposed paper is to investigate dipole-exchange spin waves propagating along the axis of a ferromagnetic nanotube in the presence of a spin current induced by a temperature gradient. Within the framework of the linearized Landau-Lifshitz equation in the magnetostatic approximation, we derive the equation for the magnetic potential of spin excitations in such a system and obtain the dispersion relation for the corresponding spin waves. Particular attention is paid to the influence of the thermoelectrically induced spin current on the spin wave dynamics, including effective damping of the spin waves and the conditions for their amplification and generation.

System of equations for a spin wave

In order to describe spin waves in the system under consideration, let us use the Landau–Lifshitz equation for the magnetization vector. In the absence of spin current and dissipation this equation can be written in the form [20]

$$\frac{\partial \mathbf{M}}{\partial t} = \gamma[\mathbf{M} \times \mathbf{H}_{\text{eff}}],$$

where γ is the gyromagnetic ratio and $\mathbf{H}_{\text{eff}}$ is the effective magnetic field inside the ferromagnet. After accounting for the magnetic dipole–dipole interaction, the exchange interaction, the magnetic anisotropy, and the external magnetic field, one can obtain the effective field in the form

$$\mathbf{H}_{\text{eff}} = \mathbf{H}^{(i)} + \alpha\Delta\mathbf{M} + \beta\mathbf{n}(\mathbf{M}\mathbf{n}).$$

Here $\mathbf{n}$ is a unit vector directed along the anisotropy axis (in our case coinciding with the unit vector $\mathbf{e}_z$).

Let us consider energy dissipation in the system by introducing the Gilbert damping term

$$\mathbf{T}_G = \frac{\alpha_G}{M}\left[\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}\right],$$

and the influence of the spin current on the magnetization dynamics – by introducing the spin-transfer torque term in the Zhang-Li form.

In contrast to earlier papers by the authors [6, 7] where the spin-polarized current was assumed to be externally applied by the electric spin-polarized current, in the present system, the spin-polarized current arises due to the Seebeck effect. In the framework of the model described in the previous section, the spin-polarized charge current density J_s can be expressed as

$$J_s = P_T J = \frac{\sigma_\uparrow S_\uparrow - \sigma_\downarrow S_\downarrow}{\sigma_\uparrow S_\uparrow + \sigma_\downarrow S_\downarrow} J = -(\sigma_\uparrow S_\uparrow - \sigma_\downarrow S_\downarrow)\nabla T, \quad (1)$$

where P_T is the thermoelectric spin-polarization parameter defined in the previous section.

In the previous papers by the authors, the influence of a spin-polarized current on the magnetization dynamics was described by the Slonczewski-Berger spin-transfer torque term [6, 7]

$$\mathbf{T}_{SB} = \frac{\varepsilon\gamma\hbar J}{2eM_0^2 d}\left[\mathbf{M} \times (\mathbf{M} \times \mathbf{e}_p)\right],$$

where J is the current density, $d$is the effective thickness of the magnetic layer, $\mathbf{e}_p$is the unit vector of spin polarization of the injected carriers, and ε is the spin-transfer efficiency. Such a form is natural for magnetic multilayers and spin-valve-type systems, where the current first passes through a polarizing magnetic layer and then enters another ferromagnetic layer whose magnetization dynamics is studied. In that situation, the torque arises due to the absorption of the transverse component of the injected spin current at the interface with the ferromagnet. In other words, only that part of the carrier spin which is not collinear with the local magnetization can transfer transverse angular momentum and produce a torque on the magnetic subsystem. If the polarization direction $\mathbf{e}_p$is parallel to the magnetization, the double vector product vanishes and the Slonczewski-Berger torque is absent.

For the nanosystem considered in the present paper, however, this physical picture is not realized. We study a single conducting ferromagnetic nanotube in which the current is generated thermoelectrically by a temperature gradient applied along the nanotube axis. Such a current is not injected from an external magnetic polarizer and does not cross an interface between two magnetic subsystems with different magnetization directions. Instead, it is generated inside the same ferromagnet in which the spin wave propagates. Since the conduction bands of a conducting ferromagnet are exchange-split, the thermoelectric current is spin-polarized already in the bulk of the nanotube. Nevertheless, this spin polarization is determined by the same local exchange field that determines the equilibrium magnetization. Therefore, in the equilibrium state of the present system, where the magnetization is directed along the nanotube axis O_z, the spin polarization of the thermoelectric current is also directed predominantly along this axis. Thus, the current flowing in the nanotube is, in the first approximation, a longitudinal spin-polarized current whose polarization is parallel to the equilibrium magnetization. Such a current does not create an interfacial transverse-spin torque of Slonczewski-Berger type. This is the principal physical reason why the use of the Slonczewski-Berger term, appropriate for the previous papers, is not justified for the present one-layer nanotube system.

Instead, the influence of the current in a single conducting ferromagnet must be described in the framework of the theory developed by Zhang and Li [21]. In this approach, the current-induced torque appears because conduction-electron spins are transported through a spatially nonuniform magnetization texture and exchange angular momentum with it. In the adiabatic approximation, the electron spin follows the local direction of magnetization while drifting with the current. If the magnetization varies in space, this transport of spin polarization leads to a convective transfer of magnetic texture and gives rise to the so-called adiabatic spin-transfer torque. In addition, because the tracking of the magnetization by the conduction-electron spin is not perfectly exact, spin relaxation and spin dephasing produce a small misalignment between the carrier spin and the local magnetization. This gives rise to the nonadiabatic spin-transfer torque, which has a different symmetry and often plays a role analogous to an additional damping-like contribution.

In terms of the magnetization vector **M**, the current-induced part of the Landau-Lifshitz equation should therefore be written as

$$\mathbf{T}_{ZL} = -(\mathbf{u} \cdot \nabla)\mathbf{M} + \frac{\xi}{M_0}[\mathbf{M} \times (\mathbf{u} \cdot \nabla)\mathbf{M}],$$

where ξ is the nonadiabaticity parameter and **u** is the spin-drift velocity. The first term is the adiabatic Zhang-Li torque, and the second term is the nonadiabatic Zhang-Li torque. In the present problem the charge current flows along the nanotube axis, so that $\mathbf{u} = u_z\, \mathbf{e}_z$. The quantity u_z is proportional to the spin-polarized part of the charge current density. It is convenient to write it in the form

$$u_z = \frac{\mu_B}{eM_0} J_s = \frac{P_T \mu_B}{eM_0} J_z,$$

where J_z is the total thermoelectric charge current density along the nanotube axis,

$$J_z = -(\sigma_\uparrow S_\uparrow + \sigma_\downarrow S_\downarrow)\frac{\partial T}{\partial z} = -\sigma S \frac{\partial T}{\partial z},$$

where σ is the total electric conductivity and S is the total Seebeck coefficient, and the spin-polarized charge current density J_s has only Oz-component:

$$J_s = j_\uparrow - j_\downarrow = -(\sigma_\uparrow S_\uparrow - \sigma_\downarrow S_\downarrow)\frac{\partial T}{\partial z} = -\sigma S P_T \frac{\partial T}{\partial z}.$$

Therefore, in contrast to the previous papers, the current enters the equations not through an interfacial polarizer vector $\mathbf{e}_p$, but through the drift velocity u_z proportional to the thermoelectric charge current inside the ferromagnet itself.

After linearizing, one can obtain the following expressions for the adiabatic and nonadiabatic term, correspondingly:

$$\mathbf{t}_{ad} = -\frac{P_T \mu_B}{eM_0} J_z \frac{\partial \mathbf{m}}{\partial z}, \qquad \mathbf{t}_{nad} = \xi \frac{P_T \mu_B}{eM_0} J_z \left[\mathbf{e}_z \times \frac{\partial \mathbf{m}}{\partial z}\right].$$

The adiabatic term leads to a drift-type modification of the dispersion law, usually interpreted as the current-induced spin-wave Doppler shift [22], whereas the nonadiabatic term modifies the dissipative part of the dynamics.

Taking into account the exchange interaction, anisotropy, dissipation, and the spin-transfer torque, as well as the relation $\mathbf{H}_0^{(i)} \approx \mathbf{H}_0^{(e)}$ (which implies from the form of the demagnetizing coefficients for a tube as long as the external magnetic field is directed along the nanotube axis), the Landau–Lifshitz equation for the magnetization perturbation can be written as

$$\frac{\partial \mathbf{m}}{\partial t} = \gamma \left[\mathbf{M}_0 \times \left(\mathbf{h} + \alpha\Delta\mathbf{m} + \beta\mathbf{n}(\mathbf{mn}) - \frac{1}{M_0^2}\left(\beta(\mathbf{M_0 n})^2 + \mathbf{M_0 H_0^{(e)}}\right)\mathbf{m}\right)\right] + \mathbf{t}_G + \mathbf{t}_{ad} + \mathbf{t}_{nad},$$

with the linearized Gilbert damping term having the following form:

$$\mathbf{t}_G = \frac{\alpha_G}{M_0}\left[\mathbf{M_0} \times \frac{\partial \mathbf{m}}{\partial t}\right].$$

Let us consider the magnetization and magnetic field perturbations in the form of harmonic oscillations:

$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}_0(\mathbf{r})e^{i\omega t},$$

$$\mathbf{h}(\mathbf{r}, t) = \mathbf{h}_0(\mathbf{r})e^{i\omega t}.$$

Substituting these expressions into the linearized Landau–Lifshitz equation and taking into account that the equilibrium magnetization is directed along the z-axis, one can obtain

$$i\omega\mathbf{m}_0 + \frac{P_T\mu_B}{eM_0}J_z\frac{\partial \mathbf{m_0}}{\partial z} = \gamma\left[M_0\mathbf{e}_z \times \left(\mathbf{h}_0 + \alpha\Delta\mathbf{m}_0 - \left(\beta + \frac{H_0^{(e)}}{M_0} - i\frac{\alpha_G}{\gamma M_0}\omega\right)\mathbf{m}_0\right) + \frac{P_T\mu_B}{e\gamma M_0^2}J_z\xi\frac{\partial \mathbf{m_0}}{\partial z}\right] \tag{2}$$

In order to obtain a closed system of equations for the magnetization and magnetic field perturbations, let us use the magnetostatic approximation. In this approximation the magnetic field of the spin wave can be represented as the gradient of a scalar magnetic potential: $\mathbf{h} = -\nabla\Phi.$ Substituting this into the Maxwell equation $\operatorname{div}\mathbf{h} = -4\pi\operatorname{div}\mathbf{m}$ leads to the following relation for the magnetic potential:

$$\Delta\Phi - 4\pi\operatorname{div}\mathbf{m} = 0.$$

For harmonic perturbations this equation takes the form

$$\Delta\Phi_0 - 4\pi\operatorname{div}\mathbf{m}_0 = 0, \tag{3}$$

here $\Phi(\mathbf{r}, t) = \Phi_0(\mathbf{r})e^{i\omega t}$.

The obtained system of equations (2) and (3) establishes the relationship between the magnetization perturbation and the magnetic potential of the spin wave. By eliminating the magnetization perturbation from these equations, one can derive the equation for the magnetic potential and obtain the sought dispersion relation for dipole–exchange spin waves in the nanotube in the presence of a thermoelectrically induced spin-polarized current.

Equation for the magnetic potential

Let us now derive the equation for the magnetic potential of the spin wave in the nanotube.

Since the system under consideration has cylindrical symmetry, it is convenient to use cylindrical coordinates (ρ, θ, z), where the z-axis coincides with the axis of the nanotube. The equilibrium magnetization $\mathbf{M}_0$, the external magnetic field $H_0^{(e)}$, and the temperature gradient are all directed along this axis.

After substitution of (3) into (2) and taking into account the relations $\mathbf{h_0} = -\nabla\Phi_0$, $J_z = -\sigma S\frac{\partial T}{\partial z}$ one obtains

$$i\omega\mathbf{m}_0 - \kappa_S\frac{\partial\mathbf{m_0}}{\partial z} = \gamma\left[M_0\mathbf{e}_z \times \left(-\nabla\Phi_0 + \alpha\Delta\mathbf{m}_0 - \left(\beta + \frac{H_0^{(e)}}{M_0} - i\frac{\alpha_G}{\gamma M_0}\omega\right)\mathbf{m}_0 - \frac{\kappa_S\xi}{\gamma M_0}\frac{\partial\mathbf{m_0}}{\partial z}\right)\right],$$

where the following parameter is introduced for the sake of convenience:

$$\kappa_S = \frac{P_T\mu_B\sigma S}{eM_0}\frac{\partial T}{\partial z}.$$

The dependence of the magnetization perturbation on the z-coordinate is assumed to be harmonic: $\mathbf{m_0}\sim\exp(-ik_{||}z)$, $\frac{\partial\mathbf{m_0}}{\partial z} = -ik_{||}\mathbf{m_0}$. Thus, the linearized form of the Landau-Lifshitz equation can be rewritten as

$$i(\omega + \kappa_S k_{||})\mathbf{m}_0 = \gamma[M_0\mathbf{e}_z \times (-\nabla\Phi_0 + \alpha\Delta\mathbf{m}_0 - \beta_k\mathbf{m_0})],$$

where the parameter

$$\beta_k = \beta + \frac{H_0^{(e)}}{M_0} - i\frac{\alpha_G}{\gamma M_0}\omega - i\frac{\kappa_S\xi}{\gamma M_0}k_{||}.$$

After some transformations analogous to the ones performed in the earlier papers by the authors [6, 7], one can obtain the equation for the magnetic potential of the spin wave in the following form:

$$\left[\frac{(\omega+\kappa_S k_{||})^2}{\gamma^2 M_0^2} - (\beta_k - \alpha\Delta)(4\pi + \beta_k - \alpha\Delta)\right]\Delta\Phi_0 + 4\pi(\beta_k - \alpha\Delta)\frac{\partial^2\Phi_0}{\partial z^2} = 0, \quad (4)$$

where the above-introduced parameter κ_S characterizes the influence of the thermoelectrically induced spin current on the magnetization dynamics.

Equation (4) describes the spatial distribution of the magnetic potential of dipole–exchange spin waves in the investigated nanosystem. It should be noted that in the absence of spin current ($\kappa_S = 0$) this equation reduces to the corresponding equation obtained earlier by the authors for dipole–exchange spin waves in cylindrical ferromagnetic waveguides (see, e.g., [23]). The presence of the thermoelectric spin current modifies the equation through the frequency shift $\omega \to \omega + \kappa_S k_{||}$, which reflects spin-wave Doppler shift observed and analyzed in current-carrying ferromagnets (see, e.g., [22]), and through a modification of the effective damping of the spin waves.

In the following section we will solve Eq. (4) in cylindrical coordinates and obtain the dispersion relation for spin waves propagating along the axis of the nanotube.

Dispersion relation and condition of spin-wave excitation

Let us obtain the dispersion relation for the investigated spin waves. Equation (4) for the magnetic potential is written in cylindrical coordinates (ρ, θ, *z*), therefore in the region $a < \rho < b$ occupied by the nanotube, it admits a solution in the form

$$\Phi_0(\rho, \theta, z) = [A_1 J_s(k_\perp \rho) + A_2 N_s(k_\perp \rho)]\exp[i(s\theta - k_\parallel z)],$$

where A_1 and A_2 are constants, J_s and N_s are the Bessel and Neumann functions of order s, respectively, s is the radial-azimuthal mode number, and $k_\perp$, $k_\parallel$ are the transverse and longitudinal wave numbers, respectively. For this function, the relations

$$\Delta\Phi_0 = -k^2\Phi_0, \qquad \frac{\partial^2\Phi_0}{\partial z^2} = -k_\parallel^2\Phi_0$$

fulfill; here, the total wave number is introduced as $k^2 = k_\perp^2 + k_\parallel^2$.

Substituting these relations into Eq. (4), we obtain the dispersion equation

$$\left[\frac{(\omega+\kappa_S k_{||})^2}{\gamma^2 M_0^2} - \Lambda(4\pi + \Lambda)\right] k^2 + 4\pi\Lambda k_\parallel^2 = 0, \tag{5}$$

where

$$\Lambda(k, k_{||}) = \beta_k + \alpha k^2 = \beta + \frac{H_0^{(e)}}{M_0} + \alpha k^2 - \frac{i}{\gamma M_0}\left(\kappa_S \xi k_{||} + \alpha_G \omega\right),$$

Thus, in the general case the dispersion relation contains two components of the wave vector. The allowed values of $k_\perp$are determined by the boundary conditions at the inner and outer surfaces of the nanotube.

In the absence of both dissipation and effective dissipation associated with the nonadiabatic term, Eq. (5) reduces to

$$\left(\omega + \kappa_S k_{||}\right)^2 = \gamma^2 M_0^2(\beta_0 + \alpha k^2)\left(\beta_0 + \alpha k^2 + 4\pi\frac{k_\perp^2}{k^2}\right), \qquad \beta_0 = \beta + \frac{H_0^{(e)}}{M_0},$$

so that for the branch with positive real frequency

$$\omega_r = -k_\parallel \kappa_S + \gamma M_0\sqrt{(\beta_0 + \alpha k^2)\left(\beta_0 + \alpha k^2 + 4\pi\frac{k_\perp^2}{k^2}\right)}.$$

The last formula corresponds to the previous papers by the authors that investigate spin waves in a nanotube without consideration of dissipation and spin current [23], but with additional term that explicitly shows the Doppler shift produced by the thermoelectric current:

$$\Delta\omega_{\mathrm{D}} = k_\parallel u_z = -k_\parallel \kappa_S.$$

Let us note that for a spin wave to exist, the damping Gilbert damping constant α_G should not exceed approximately 0.2. Generally, its value for a typical ferromagnet nanosystem used in experiments with a spin-polarized current varies approximately from 0.02 to 0.2, see, e.g., [24, 25]. The nonadiabaticity parameter ξ can also be considered small for typical cases. Thus, addends that contain product $\alpha_G\xi$ as well as quadratic by α_G or ξ addends can be omitted in the dispersion relation. Retaining only terms linear in the small parameters α_G and ξ, and representing the frequency as $\omega = \omega_r + i\omega_i$, one can obtain from Eq. (5) after a set of transformations

$$\omega_r = -\kappa k_{||}\frac{\partial T}{\partial z} + \gamma M_0\sqrt{(\beta_0 + \alpha k^2)\left(\beta_0 + \alpha k^2 + 4\pi\frac{k_\perp^2}{k^2}\right)}, \quad (6)$$

$$\omega_i = -\frac{\beta_0 + \alpha k^2 + 2\pi\frac{k_\perp^2}{k^2}}{\sqrt{(\beta_0+\alpha k^2)\left(\beta_0+\alpha k^2+4\pi\frac{k_\perp^2}{k^2}\right)}}\left(\alpha_G\omega_r + \kappa\xi k_{||}\frac{\partial T}{\partial z}\right) \quad (7)$$

here $\kappa = \frac{P_T\mu_B\sigma S}{eM_0}$. The first of these two equations shows that the adiabatic current contribution changes only the real part of the frequency and manifests itself as a Doppler shift. The second equation shows that the nonadiabatic contribution changes the effective damping of the spin wave. Depending on the sign of the product $\xi\kappa_S k_{||}$, the thermoelectric current may either increase or decrease the damping, as anticipated in the previous section.

In particular, the condition of compensation of the intrinsic Gilbert damping is

$$\alpha_G\gamma M_0\sqrt{(\beta_0 + \alpha k^2)\left(\beta_0 + \alpha k^2 + 4\pi\frac{k_\perp^2}{k^2}\right)} - (\alpha_G - \xi)\kappa_S k_{||} = 0. \quad (8)$$

When this relation is satisfied, the effective damping vanishes. If the temperature gradient is increased further in the direction for which

$$\alpha_G\gamma M_0\sqrt{(\beta_0 + \alpha k^2)\left(\beta_0 + \alpha k^2 + 4\pi\frac{k_\perp^2}{k^2}\right)} - (\alpha_G - \xi)\kappa_S k_{||} < 0, \quad (9)$$

the effective damping becomes negative and the spin wave is excited. Correspondingly, in the values' interval of the temperature gradient for which the relation

$$\alpha_G\gamma M_0\sqrt{(\beta_0 + \alpha k^2)\left(\beta_0 + \alpha k^2 + 4\pi\frac{k_\perp^2}{k^2}\right)} - (\alpha_G - \xi)\kappa_S k_{||} > 0$$

fulfils, the effective damping increases compared to the case of the absence of the temperature gradient.

Since $\kappa_S = \frac{P_T\mu_B\sigma S}{eM_0}\frac{\partial T}{\partial z}$, the threshold value of the temperature gradient is determined by

$$\left(\frac{\partial T}{\partial z}\right)_{cr} = \frac{e\alpha_G\gamma M_0^2}{P_T\mu_B\sigma S(\alpha_G-\xi)}\frac{\sqrt{(\beta_0+\alpha k^2)\left(\beta_0+\alpha k^2+4\pi\frac{k_\perp^2}{k^2}\right)}}{k_{||}} \tag{10}$$

where the sign of the gradient must be chosen so that the induced current compensates the Gilbert damping.

Therefore, the thermoelectric spin-polarized current modifies the spin-wave spectrum in two ways. The adiabatic Zhang-Li term produces the Doppler shift $\kappa_S k_{||}$, whereas the nonadiabatic term changes the effective damping and under condition (8) may compensate the intrinsic dissipation. Under condition (9), it leads to spin-wave excitation. This is the principal difference between the present thermoelectric current-driven case and the previously considered Slonczewski-type excitation mechanism.

Orthogonal wavenumber spectrum

The spectrum of the transverse (orthogonal) wavenumber $k_\perp$for dipole-exchange spin waves in a round ferromagnetic nanotube can be obtained from the boundary conditions for the magnetic potential and magnetic field on the inner and outer cylindrical surfaces of the nanotube. A detailed derivation of the corresponding spectral equation for a round nanotube in the magnetostatic approximation was given earlier by the author in Ref. [26]. In the present paper, it is not necessary to repeat that derivation in full, since the physical basis of the calculation remains the same.

Indeed, in the model considered here the thermoelectric spin-polarized current enters the dynamical equation through the bulk Zhang-Li terms, i.e. through the longitudinal drift correction and the nonadiabatic damping-like correction. These terms modify the bulk equation for the spin wave and, correspondingly, the dispersion relation through the effective frequency shift and the effective damping parameter. At the same time, within the adopted approximation, they do not change the form of the magnetostatic boundary conditions on the inner and outer cylindrical surfaces of the nanotube. Therefore, the procedure used earlier for obtaining the transverse wavenumber spectrum remains applicable to the present case as well.

For a round nanotube, the exact spectrum of $k_\perp$is determined by the corresponding boundary-condition equation written in terms of the Bessel and Neumann functions [26]. In the thin-wall approximation, this equation is substantially simplified. If the nanotube wall thickness is small compared to the inner radius, so that $(b-a)/a \ll 1$, and the longitudinal wavenumber is small compared to the transverse one, so that $k_\parallel \ll k_\perp$ (which fulfils on the most part of the $k_\parallel$ values' interval, since $k_\parallel$ must be greater than or of the same order with the reciprocal length of the nanotube, while $k_\perp$ for non-zero modes must be greater than or of the same order with $(b-a)^{-1}$), then the dependence of the transverse spectrum on $k_\parallel$ is weak and the transverse wavenumber is determined, in the first approximation, by the quasi-one-dimensional standing-wave condition across the nanotube wall [26]:

$$k_\perp(b-a) = \pi n, \qquad n = 1, 2, 3, \dots$$

Thus,

$$k_{\perp} = \frac{\pi n}{b-a} \tag{11}$$

The fundamental mode $n = 0$ should be treated separately from the physical point of view (as the mode approximately uniform across the wall thickness, for which $k_{\perp} = 0$ – while nonzero modes $n > 0$ are described by the thin-wall standing-wave model). As the expression (11) formally fulfils when $n = 0$ (fundamental mode), the expression (11) with n starting with 0 will be used in the following paper section – however, as we will see form the analysis, the spin wave pattern is essentially different for the fundamental mode compared to the higher modes.

Hence, in the present problem the thermoelectric spin current modifies the real and imaginary parts of the spin-wave frequency, but does not change the leading-order form of the transverse quantization rule. The latter remains the same as for the round nanotube without spin current, provided the above conditions are fulfilled and the spin current is not strong enough to change the equilibrium magnetic configuration of the nanotube.

Results and discussion

Let us analyze the obtained results.

As it has already been mentioned before, the above-obtained dispersion relation shows that the thermoelectrically induced spin-polarized current influences the spin-wave spectrum in two qualitatively different ways. First, the adiabatic Zhang-Li term produces a shift of the real part of the frequency through the replacement $\omega \to \omega_{\mathrm{eff}} = \omega + \kappa_S k_{\parallel}$. Thus, the thermoelectric current causes a current-induced Doppler shift of the spin-wave spectrum. This result is consistent with the general physical picture of spin-wave Doppler shift in conducting ferromagnets carrying a spin-polarized current [22]. Let us note that the sign of this shift is determined by the sign of the product $\kappa_S k_{\parallel}$. Therefore, spin waves propagating in opposite directions along the nanotube axis become nonequivalent in the presence of the temperature gradient. In this sense, the thermoelectric current removes the symmetry between the modes with $+k_{\parallel}$ and $-k_{\parallel}$.

Second, the nonadiabatic Zhang-Li term enters the imaginary part of the frequency together with the Gilbert damping term. As a result, the temperature gradient affects not only the phase velocity of the spin wave, but also its attenuation. Depending on the sign of the product $\kappa_S k_{\parallel}$, the nonadiabatic contribution may either partially compensate the intrinsic dissipation or increase it. Hence, the temperature gradient may either facilitate spin-wave excitation or suppress the propagation of already existing modes. In contrast to the adiabatic term, which influences only the real part of the frequency, the nonadiabatic term acts as a damping-like correction.

A special case corresponds to the point $\alpha_G = \xi$. Current-dependent term in the imaginary part of the frequemcy is proportional to $(\alpha_G - \xi)\kappa_S k_{\parallel}$. Therefore, at $\alpha_G = \xi$ the contribution of the thermoelectric spin current to the effective damping vanishes, and the imaginary part of the frequency reduces to the ordinary damping term,

$$\omega_i = -\left(\beta_0 + \alpha k^2 + 2\pi \frac{k_\perp^2}{k^2}\right),$$

in the adopted normalized notation. Correspondingly, the critical temperature gradient is proportional to $1/(\alpha_G - \xi)$ and tends to $+\infty$ or $-\infty$ (when $\alpha_G \to \xi$) from opposite sides. Thus, the point $\alpha_G = \xi$ separates two qualitatively different regimes of the influence of the thermoelectric current on spin-wave damping: on one side of this point the current decreases the damping, whereas on the other side it increases it. Exactly at $\alpha_G = \xi$, the thermoelectric current produces only the Doppler shift of the real part of the frequency and does not affect the damping.

An important limiting case is the absence of the temperature gradient, so $\kappa_S = 0$. Then, the obtained expressions reduce to the previously known results for dipole-exchange spin waves in a ferromagnetic nanotube without spin current. In turn, if the spin current is present but the nonadiabaticity parameter is neglected, the temperature gradient affects only the real part of the frequency and does not change the attenuation of the wave. Thus, the roles of the adiabatic and nonadiabatic terms are clearly separated in the analytical results.

The obtained expressions also make it possible to identify the conditions under which the effect of the thermoelectric current is most pronounced. Since the correction to the real part of the frequency is proportional to $k_\parallel$, the Doppler shift vanishes for purely transverse standing modes and increases with increasing longitudinal wave number. However, the relative contribution of this shift to the total frequency decreases in the short-wave exchange-dominated regime, where the main part of the frequency grows approximately as k^2. Therefore, the relative influence of the thermoelectric current on the dispersion law is expected to be most visible in the intermediate region of wave numbers, where the exchange and magnetostatic contributions are comparable.

A similar conclusion follows for the condition of spin-wave excitation, but only for the fundamental mode. The threshold temperature gradient is determined by the condition of compensation of the Gilbert damping by the nonadiabatic current-induced term, see (10). Its dependence on $k_{||}$ features a pole in the point $k_{||} = 0$, in the vicinity of which $(\partial T/\partial z)_{cr} \sim const/k_{||}$. Analysis shows that with increasing of $|k_\parallel|$, for the fundamental mode ($n = 0$) $|(\partial T/\partial z)_{cr}|$ reaches minimum value and then start increasing, but for the higher modes the dependence becomes monotonous instead. Hence, the excitation of spin waves by the thermoelectric current is expected to be most efficient for the short waves at the higher modes ($n > 0$). However, for the fundamental mode, this excitation is expected to be most efficient not for the longest or shortest waves, but for modes with finite $k_\parallel$.

It is also important that, within the adopted approximation, the thermoelectric current modifies the frequency and damping of the spin wave but does not change the leading-order quantization rule for the transverse wavenumber. Therefore, the transverse mode structure of the nanotube is still determined primarily by its geometrical parameters, whereas the temperature gradient controls the propagation and attenuation of these modes along the nanotube axis. This means that the geometrical quantization and the thermoelectric current-induced effects can be treated as two relatively independent mechanisms of spin-wave spectrum formation.

Let us make graphical representations of the obtained results in the absence of the external magnetic field ($H_0^{(e)} = 0$, $\beta_0 = \beta$) for a nanotube with the thickness *b*-*a* = *d* = 10 nm. From the expression for the orthogonal spectrum (11) implies that for the condition $k_{\|} \ll k_{\perp}$ to fulfill, the values of $k_{||}$ should be limited by approximately 10^6 cm^{-1}. Then, if the $Ni_{80}Fe_{20}$ ferromagnet is chosen as the nanotube material (its parameters for the graphs below are taken from the papers [27–29]), the dependencies of the real and imaginary parts of the spin wave frequencies (defined by the dispersion laws (6), (7)) on the longitudinal wavenumber $k_{\|}$ and the temperature gradient $\frac{\partial T}{\partial z}$ for the first three transversemodes (*n* = 0, *n* = 1 and *n* = 2) are as follows (Figs. 2,3):

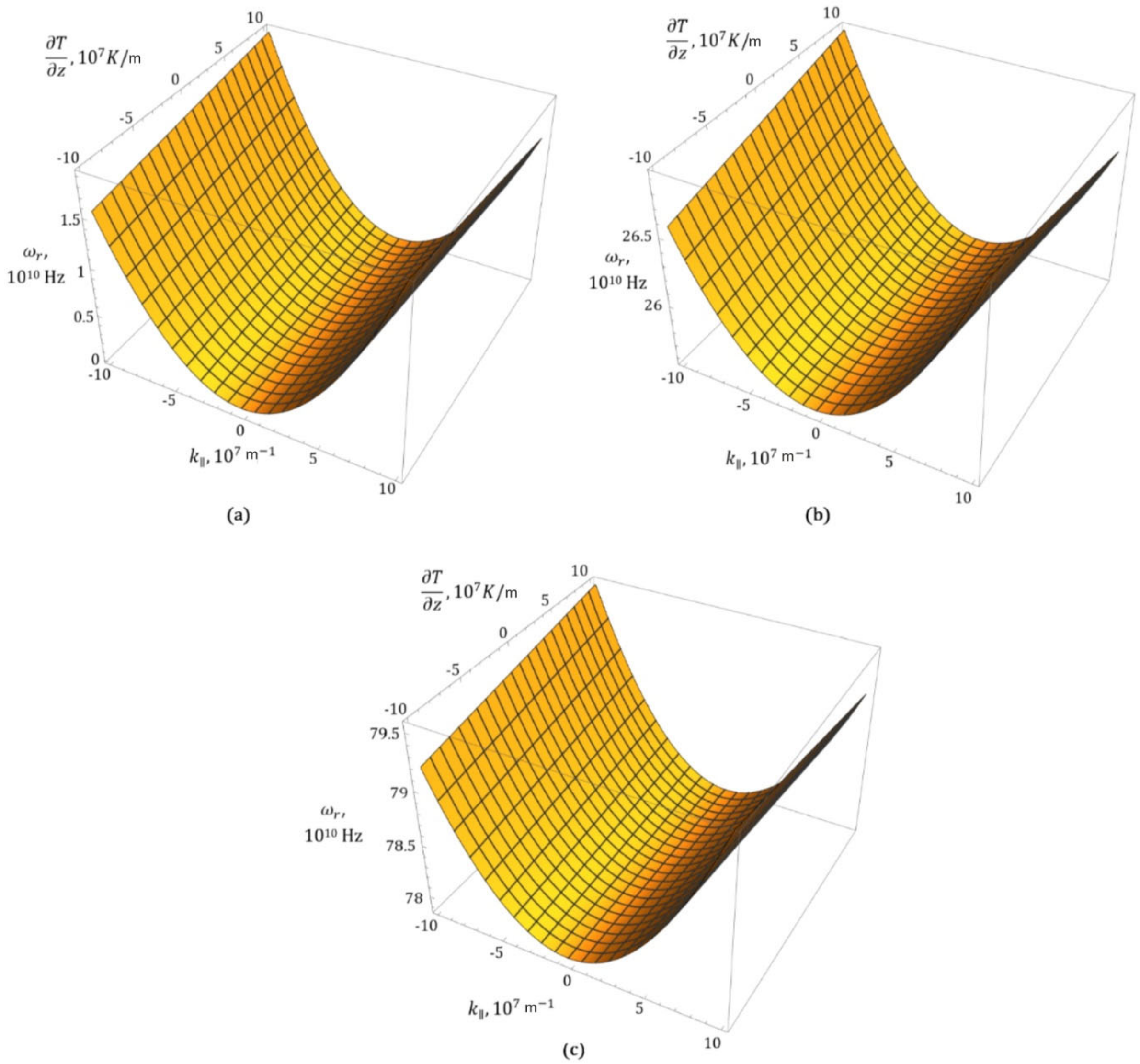


Fig. 2. Dependencies $\omega_r(k_{\|}, \frac{\partial T}{\partial z})$ for the nanotube material $Ni_{80}Fe_{20}$, nanotube thickness d = 10 nm and (a) n = 0, (b) n = 1, (c) n = 2

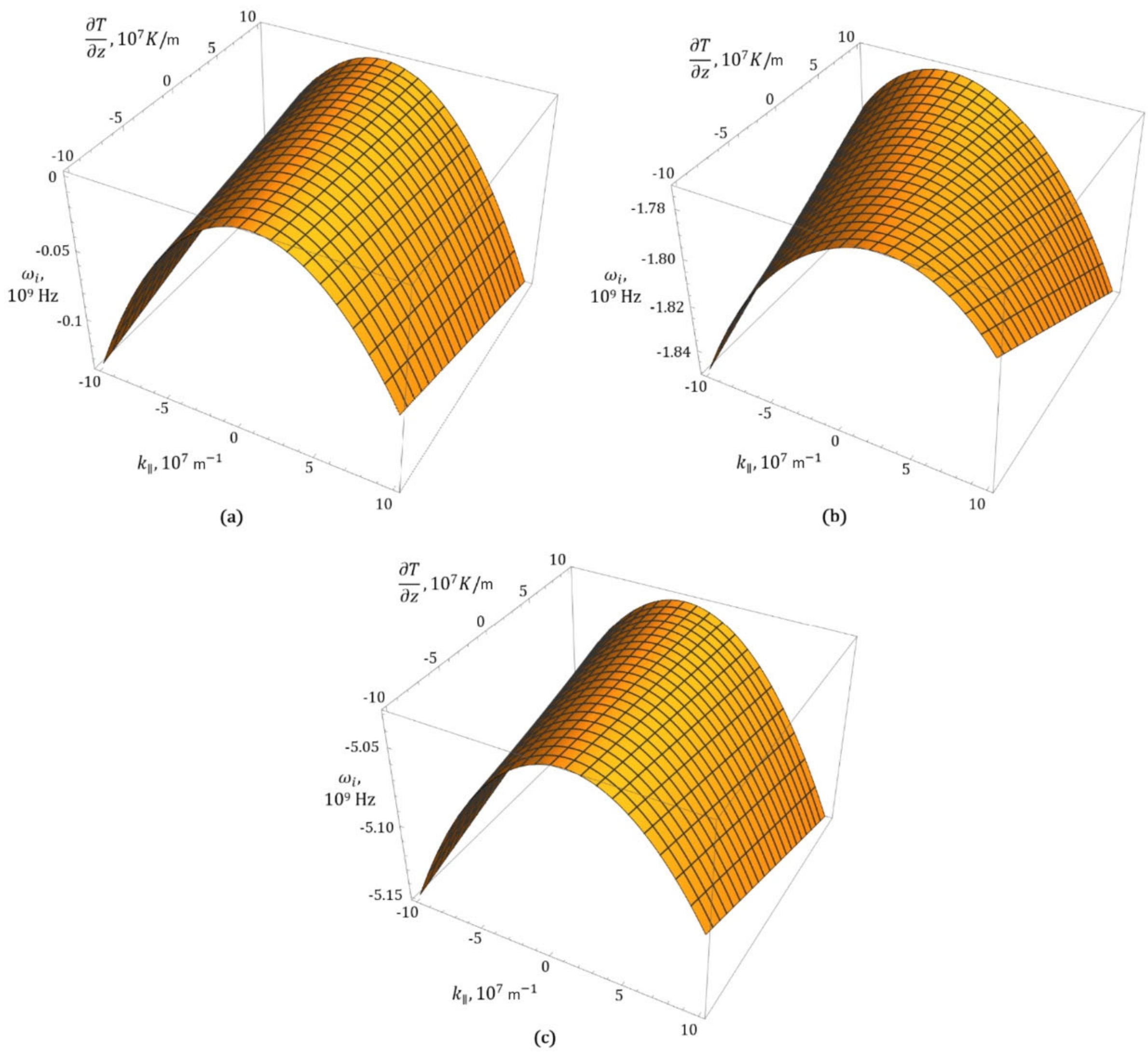


Fig. 3. Dependencies $\omega_i(k_\parallel, \frac{\partial T}{\partial z})$ for the nanotube material $Ni_{80}Fe_{20}$, nanotube thickness d = 10 nm and (a) n = 0, (b) n = 1, (c) n = 2

As it can be seen from the Figs. 2, 3, dependencies for both real and imaginary parts of the frequency on $k_\parallel, \frac{\partial T}{\partial z}$ have similar looks for different transverse modes, but on essentially different scales (within two orders of magnitude for these three modes – and that's why they have been displayed on separate graphs for each transverse mode). There are pronounced dependencies on $\frac{\partial T}{\partial z}$ (everywhere except for the region where $k_\parallel$ is close to 0), which shows that the thermoelectrically induced spin-polarized current can noticeably modify both the real and imaginary parts of the spin-wave frequency in the considered parameter range. The dependencies on $\frac{\partial T}{\partial z}$ are linear, but not symmetric with respect to inverting the sign of this value. The dependencies on $k_\parallel$, however, are not linear. For ω_r, they are close to parabolic (in fact, a sum of linear function and a function that is close to parabolic, but the linear part is weakly pronounced for the used set of the ferromagnet parameters), with the minimal value

$\omega_r = \beta\gamma M_0 = 0.17$ GHz (for the used set of the ferromagnet parameters) achieved when $n = 0$, $k_\parallel = 0$. Dependencies $\omega_i(k_\parallel)$ are close to monotonous, having weakly pronounced maximum. Analysis of the Eq. (7) shows that for $n = 0$, this maximum takes place when

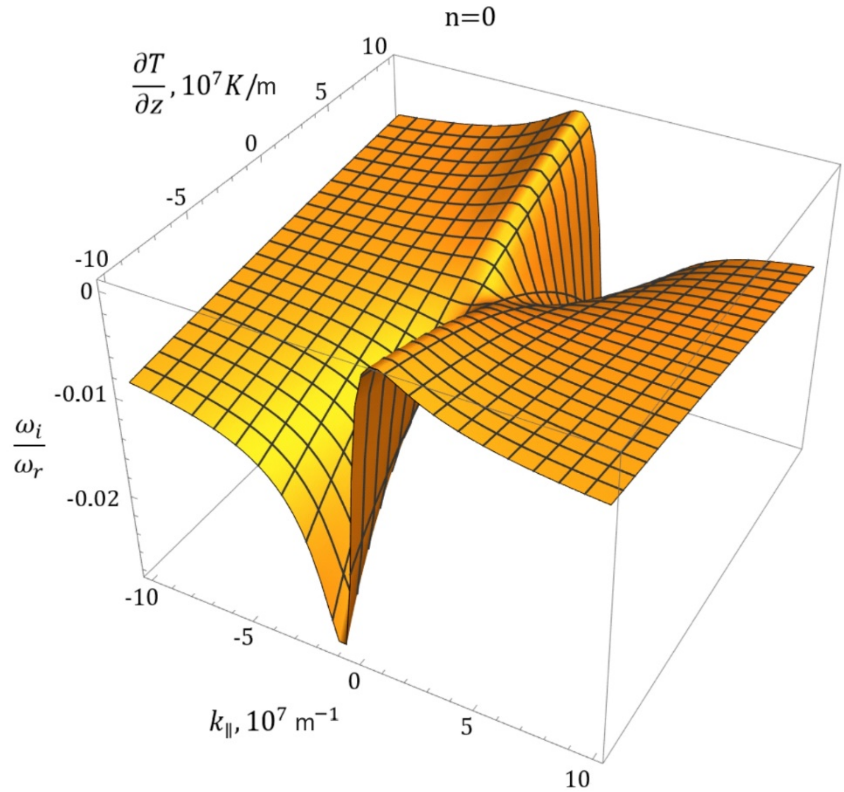
$$k_\parallel = k_{||}^{(max)} = \frac{\kappa(\alpha_G - \xi)}{2\alpha_G \gamma M_0 \alpha} \frac{\partial T}{\partial z},$$

and for $n > 0$ the maximum condition (in the same practically important limit $k_\parallel \ll k_\perp$) takes form

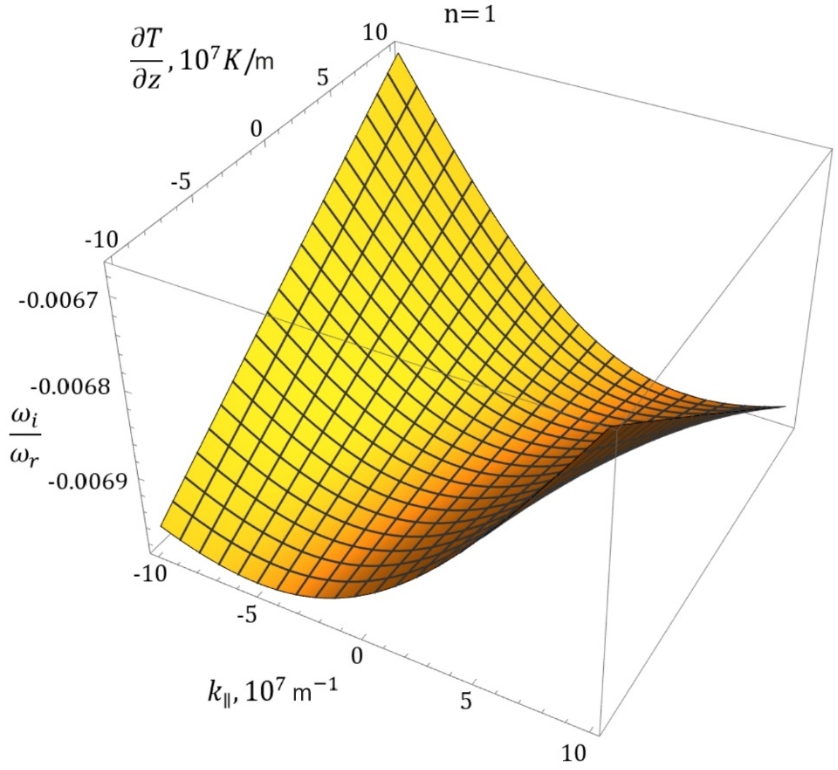
$$k_\parallel = k_{||}^{(max)} \approx \frac{\kappa(\alpha_G - \xi)}{2\alpha_G \gamma M_0 \left(\alpha - \frac{2\pi}{k_\perp^2}\right)} \frac{\beta + \alpha k_\perp^2 + 2\pi}{\sqrt{(\beta + \alpha k_\perp^2)(\beta + \alpha k_\perp^2 + 4\pi)}} \frac{\partial T}{\partial z}.$$

Let us note that on the condition $\alpha k_\perp^2 - 2\pi < 0$, the extremum given by the above formula is a minimum, not a maximum. In that case one has to retain higher-order terms in $k_\parallel / k_\perp$ when analyzing Eq. (7) for the extremum condition.

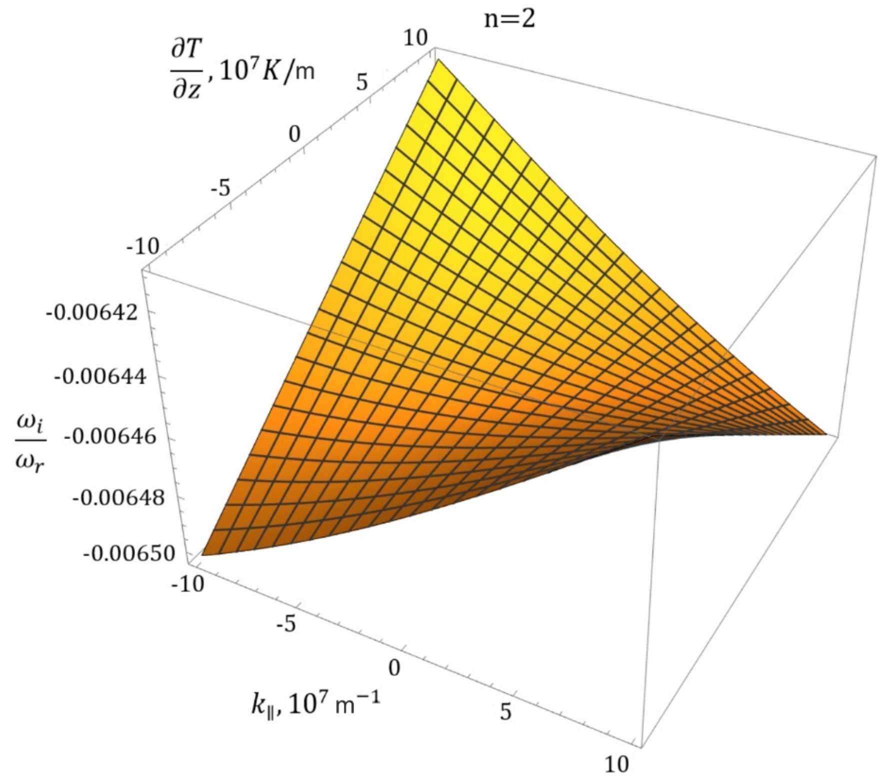
Dependencies $\frac{\omega_i}{\omega_r}(k_\parallel, \frac{\partial T}{\partial z})$ for the same nanotube configuration are represented on the Fig. 4.



(a)

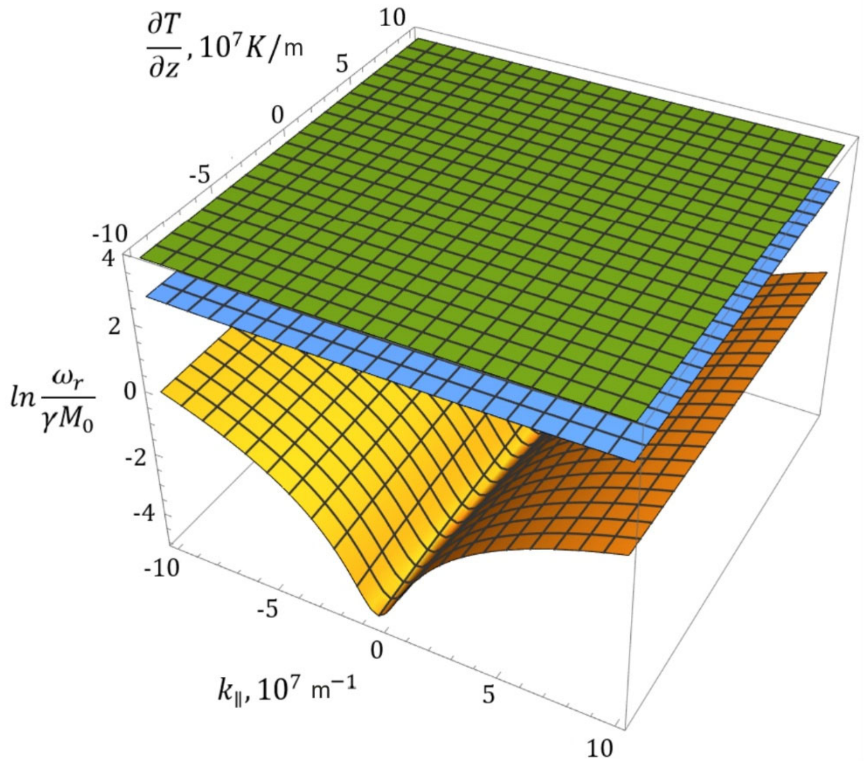


(b)

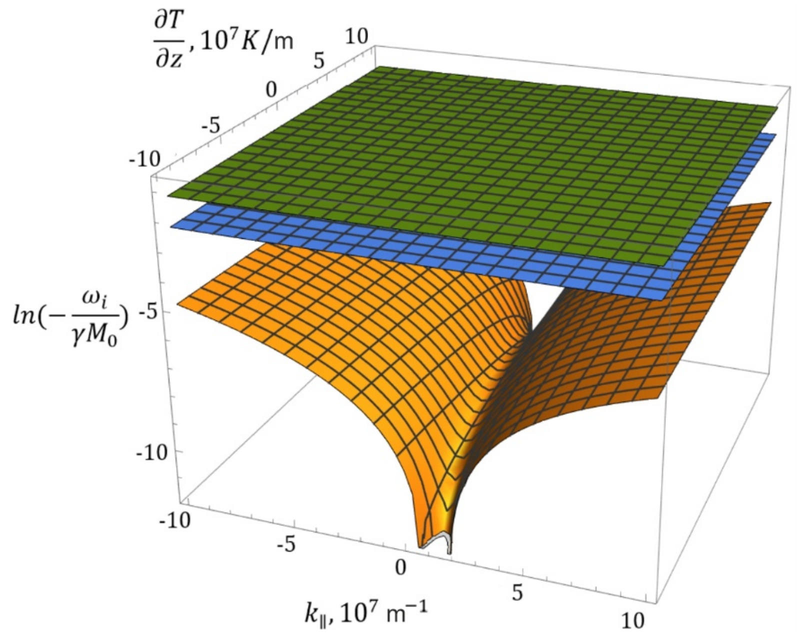


(c)

Fig. 4. Dependencies $\frac{\omega_i}{\omega_r}(k_\parallel, \frac{\partial T}{\partial z})$ for the nanotube material $Ni_{80}Fe_{20}$, nanotube thickness d = 10 nm and (a) n = 0, (b) n = 1, (c) n = 2



(a)



(b)

Fig. 5. Logarithmic dependencies a) $ln\frac{\omega_r}{\gamma M_0}(k_\parallel, \frac{\partial T}{\partial z})$ and b) $ln(-\frac{\omega_i}{\gamma M_0}(k_\parallel, \frac{\partial T}{\partial z}))$
for the nanotube material $Ni_{80}Fe_{20}$ and nanotube thickness d = 10 nm

As one can see from the Fig. 4, the dependencies $\frac{\omega_i}{\omega_r}(k_\parallel, \frac{\partial T}{\partial z})$ for the zero mode $n = 0$ and higher modes $n > 0$ differ qualitatively. Modes with $n = 1$ and $n = 2$ have similar form; while they both look like saddle surfaces, in fact the dependencies on $\frac{\partial T}{\partial z}$ for both of them are monotonous, close to linear ones. The dependencies on $k_{\|}$ for higher modes ($n > 0$) contain single global minimum each. The dependence for the fundamental mode $n = 0$ turns out to be much more complex – with dependence on $k_{\|}$ having one local maximum and one local minimum at the points $k_{||} = \pm\sqrt{\beta/\alpha}$ (except for the case $\frac{\partial T}{\partial z} = 0$ for which $\frac{\omega_i}{\omega_r} = -\alpha_G = const$), asymptotically approaching the value $-\alpha_G$ for big in absolute $k_{||}$ (both positive and negative).

As it can be seen from the Fig. 5, visualizing logarithmic variations of the dependencies $\omega_r(k_{\|}, \frac{\partial T}{\partial z})$, $\omega_i(k_{\|}, \frac{\partial T}{\partial z})$ for different transverse modes on a single graph (for each part of the frequency) allows to visualize the characteristic scale differences for different modes but not differences of the dependencies themselves. Because of the characteristic scale differences, only the dependencies for the mode $n = 0$ look distinguishable while dependencies for $n = 1$ and $n = 2$ look planar. (In normal, non-logarithmic visualizations dependencies for all three modes appear planar).

Conclusions

Thus, a theoretical study of dipole-exchange spin waves in a conducting ferromagnetic nanotube with the easy axis directed along the nanotube axis has been carried out in the current paper. A temperature gradient is assumed to be applied along the nanotube, generating a thermoelectrically induced spin-polarized current. In contrast to the authors' previous papers [6,7] (in which different source of a spin current has been considered), the interaction of the current with the spin-wave subsystem was described in the physically consistent Zhang-Li form, which is appropriate for a single conducting ferromagnet in which the current is generated inside the same magnetic body. Within this approach, the adiabatic and nonadiabatic current-induced terms were introduced into the Landau-Lifshitz equation, and the corresponding equation for the magnetic potential, dispersion relation, and spin-wave excitation conditions were obtained.

It has been shown that the thermoelectric spin-polarized current influences the spin-wave spectrum in two qualitatively different ways. The adiabatic current contribution leads to a Doppler shift of the real part of the spin-wave frequency and, therefore, removes the equivalence of waves propagating in opposite directions along the nanotube axis. The nonadiabatic contribution modifies the imaginary part of the frequency and, depending on the direction and magnitude of the temperature gradient, may either increase the effective damping or partially compensate it, or lead to a spin wave generation. Thus, the temperature gradient acts as a control parameter not only for the spin-wave phase velocity, but also for the attenuation and generation of spin waves.

It has also been established that the compensation of the intrinsic Gilbert damping by the nonadiabatic current-induced term determines the threshold value of the temperature gradient required for spin-wave excitation. A special case corresponds to the point $\alpha_G = \xi$, at which the effective damping vanishes, while the Doppler shift of the real part of the frequency remains finite. Therefore, this point separates two qualitatively different regimes of the influence of the thermoelectric current on spin-wave damping.

The analysis of the transverse spectrum has shown that, within the adopted approximation, the thermoelectric current does not change the leading-order quantization rule for the orthogonal wavenumber. Hence, the transverse mode structure of the nanotube remains determined mainly by its geometry, whereas the temperature gradient governs the longitudinal propagation and attenuation of the modes. In the thin-wall approximation, the higher transverse modes obey the usual quasi-one-dimensional standing-wave condition across the nanotube wall, while the fundamental mode should be treated separately as a mode approximately uniform across the thickness.

The numerical analysis performed for a permalloy nanotube has demonstrated that the current-induced modifications of the spectrum are most pronounced for the fundamental mode. For all investigated modes, the real part of the frequency exhibits a clear Doppler-type dependence on the temperature gradient, while the imaginary part reveals the possibility of damping control and spin-wave excitation. At the same time, the characteristic scales of the frequency strongly differ for different transverse modes, which indicates that the thermoelectric current affects the low-lying and higher modes in quantitatively different ways. Values' interval for the real part of the spin wave frequency starts from 0.17 GHz and reaches THz region for higher (non-fundamental) spin wave modes.

Thus, the obtained results show that a temperature gradient in a conducting ferromagnetic nanotube can serve as an efficient means of controlling the spectrum, nonreciprocity, damping, and excitation of dipole-exchange spin waves. This makes the considered nanosystem promising for thermally controlled magnonic devices, in which spin-wave transport and generation can be tuned by thermoelectric means without the need for an external spin injector.

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Теорія дипольно-обмінних спінових хвиль у феромагнітній нанотрубці за наявності термоелектрично індукованого спін-переносного моменту

В роботі представлено теоретичне дослідження дипольно-обмінних спінових хвиль у провідній феромагнітній нанотрубці, що перебуває під дією поздовжнього температурного градієнта. Температурний градієнт завдяки ефекту Зеєбека генерує електричний струм, який у феромагнетику стає спін-поляризованим та зумовлює виникнення спін-переносного моменту, що діє на намагніченість. Динаміка намагніченості описується в межах формалізму Ландау–Ліфшиця–Гільберта, доповненого членами спін-переносного моменту у формі Zhang–Li, які відповідають адіабатичному та неадіабатичному спін-переносу. Отримано аналітичне дисперсійне співвідношення для спінових хвиль у тонкостінній нанотрубці з одновісною магнітною анізотропією з урахуванням як обмінної, так і диполь-дипольної взаємодій. Показано, що термоелектрично індукований спін-поляризований струм призводить до появи доплероподібного зсуву у спектрі спінових хвиль та змінює ефективне загасання. Визначено критичне значенння температурного градієнту, за якого неадіабатичний спін-переносний момент компенсує власне гільбертівське загасання, з подальшою генерацією спінових хвиль. Чисельні оцінки для нанотрубок з пермалою показують, що зазначений ефект може бути суттєвим за експериментально досяжних параметрів. Отримані результати виявляють прямий зв'язок між термоелектричним переносом заряду та динамікою спінових хвиль у викривлених магнітних наноструктурах, що підкреслює потенціал феромагнітних нанотрубок як елементів спін-калоритронних та термоелектричних пристроїв.

Ключові слова: спінова хвиля; наномагнетизм; дипольно-обмінна теорія; феромагнітна нанотрубка; одновісна магнітна анізотропія; спін-поляризований струм; термоелектрика; спін-перенос; ефект Зеєбека.